Predictive Centroiding for Star Trackers with the Effect of Image Smear

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Abstract

Star centroiding, locating the star center in a star image frame, is a fundamental process for any star tracker. In this paper, the approximate locations of the stars in successive image frames are predicted using the angular velocity as provided by a rate gyro, then the centroid is updated based upon local image processing. When the rate gyro data are not available, then the angular velocity is estimated using the attitude kinematics equation and successive attitude estimates from the Lost-In-Space Algorithm. Also considered are the special features of noncircular star image shapes associated with optical tagging of starlight and/or image smear. Finally, an approach is presented to implement these ideas with the recently introduced Active Pixel Sensors, allowing dynamic pixel access and selected subarray analog-to-digital conversion of the pixel information is feasible, with logic dictated by most recent image and the instantaneous angular velocity estimate. This novel process that predicts the star image starlight locations is termed predictive centroiding. The problem of the image smear is also treated, in which the relatively high angular velocity of the spacecraft will affect the shape of the star images. These approaches, coupled with active pixel sensors, should enable near-optimal image processing and high frame rates. The paper includes analytical, computational, and night sky experimental results.

Introduction

Star trackers are widely used in spacecraft attitude determination because they produce higher accuracy attitude estimates than any other existing sensors. In order to maximize the accuracy of the star direction estimation, the starlight is usually defocused over $3 \times 3$ to $15 \times 15$ pixel array masks, depending on the sensor [e.g.,

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1 Paper AAS 02-103 presented at the AAS/AIAA Space Flight Mechanics Meeting, San Antonio, TX, 27–31 January, 2002. The presented version did not have the night sky experiments included in the present manuscript.
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Charge Coupled Devices (CCD) design and/or on the star magnitude. The resulting shape of the CCD starlight spot is usually described by Point Spread Functions (PSFs) [1] which are typically near-Gaussian. The appropriate optical designs defocus the starlight and utilizing modern image centroiding techniques allows estimating the star direction with a precision of 1/10 of a pixel or better.

Image centroiding is a fundamental process to optimize the attitude data set accuracy. Recently, with the introduction of multiple field-of-view star trackers [2] (which output noncircular images to identify the associated field of view), the centroiding algorithms have new challenges to meet. The speed and accuracy to accomplish the centroiding process represent, therefore, the important measures to compare the existing and new approaches. The frame rate of star trackers using an advanced Active Pixel Sensor (APS) can be relatively high (10 to 100 Hz), and for typical small optics and associated realistic integration times, the maximum angular velocity will necessarily be fairly low (significantly less than one degree/sec). In view of these considerations, a typical star will be imaged many successive times (typically, several hundred times) before it leaves the field of view. In successive image processing, to enable local access and analog-to-digital conversion of only those pixels where starlight is likely to be found, we can make use of previous star locations and the approximately linear displacement of the star locations from one frame to another one.

In many applications, if \( \omega \) is the angular velocity vector and the \( \delta t \) is the time interval between successive frames then we can simply assume \( \omega \delta t \) is the differential angular displacement of the star sensor and map this rotation into linearly predicted displacements of the image centers for all stars. This linear approximation is usually more than adequate to center the starting adaptive mask for computing star centroids, since the small errors in the starting approximation for centering the mask simply accelerate the process of finding the data to centroid; using this approach, there is usually negligible effect on the final computed centroid approximation. The predictive centroiding algorithm is implemented to be used in the GIFTS EO-3 (Geostationary Imaging Fourier Transform Spectrometer) mission. Figure 1 provides a schematic of the split FOV camera that will be used in the GIFTS mission. The electro-optical details of this camera design are the subject of a pending patent.
[2], however, the essential truth is that two orthogonal star fields are simultaneously imaged onto the same focal plane detector.

Predictive Centroiding Steps

Once we have the star image, taken by one or multiple FOV cameras, the centroiding process is needed. Initially, the image will be treated using the usual centroiding techniques which use the mass moment method to find the location of the star in the image. This is the most common approach and has been motivated by the acquisition and tracking algorithms developed historically for the ASTROS star tracker developed at NASA JPL [3]. After the first image, the following images will be processed using our new technique, predictive centroiding. Predictive centroiding will be treated for the one, two and three Field Of View (FOV) star trackers [2, 4, 5]. The steps of predictive centroiding are stated as follows:

1. The attitude matrix (projecting body frame directions onto the inertial frame), $C(t_0)$ evaluated at initial time, is calculated using the Lost In Space Algorithm (LISA) [7].

2. Using the initial angular velocity data, we can predict the attitude matrix at the current frame $C(t + \delta t)$ using the previous attitude matrix $C(t)$, and the following linear approximation

$$
C(t + \delta t) = [I - \vec{\omega} \delta t] C(t)
$$

where $\vec{\omega}$ is the cross product matrix populated with the components of the angular velocity vector $\vec{\omega}$. For virtually all current anticipated missions, $\|\vec{\omega} \delta t\| < 10^{-3} \text{ rad.}$, so equation (1) should be accurate to micro radian or better precision for a one time step prediction (the angle change). Since the inertial attitude errors themselves are typically $5 \mu$ radians or larger, the one-step predictions (angle change) error is near negligible as regards restarting the centroiding algorithm.

3. The vectors associated with the four corners of the FOV are projected to the inertial reference frame using the predicted attitude matrix.

4. By accessing the star catalog using a bounding box whose vertices are the sensor four corners, we can access the inertial reference vectors $\hat{\vec{v}}_i$ to stars imaged in that frame.

5. Given the attitude matrix $C(t) = [\hat{\vec{e}}_1 \hat{\vec{e}}_2 \hat{\vec{e}}_3]$ at time $t$, and the inertial star vectors $\hat{\vec{v}}_i$, ($i = 1, \ldots, n$), the start locations $(x_i, y_i)$ are evaluated by the colinearity equations

$$
x_i = x_0 - f \frac{\hat{\vec{e}}_2^T \hat{\vec{v}}_i}{\hat{\vec{e}}_3^T \hat{\vec{v}}_i} \quad \text{and} \quad y_i = y_0 - f \frac{\hat{\vec{e}}_3^T \hat{\vec{v}}_i}{\hat{\vec{e}}_3^T \hat{\vec{v}}_i}
$$

where $f$ is the camera focal length and $x_0, y_0$ are the optical axis offsets, which are determined by using the ground calibration [9].

6. For the considered frame, once the star centroid locations are predicted, these locations become the center of masks used for centroiding for the real CCD image.

7. The recursive star identification algorithm [6], which uses the star neighbor approach, will then be used to identify the observed stars for that frame. The optimal estimate of the attitude matrix $C(t + \delta t)$ will be determined by using the
ESOQ-2 method \[8\], and the associated angular velocity estimate will be calculated from the attitude kinematics equation \[
\frac{dC}{dt} = -\tilde{\omega}C
\]
by replacing differentials with small finite differences
\[
\tilde{\omega} \equiv \left[ I - C(t + \delta t)C^T(t) \right] / (\delta t)
\] (3)

Alternatively, we can use measured angular velocity from rate gyros, if available.

8. The angular velocity measured by the rate gyros (or estimated by equation (3)) will be used to determine the predicted star locations at the next frame and the loop will start again from Step 2.

Some simulation results are shown in Figs. 2 and 3, for one FOV camera where two sequential images are superimposed and the two sets of star locations are shown to indicate the image motion. Also, for the case of two superimposed orthogonal FOVs of a split field of view camera (StarNav II), Figs. 4 and 5 show the star images and the star locations for two adjacent-in-time images. The angular velocity rates are assumed for GEO motion with 0.05 deg/sec frequency and 0.5 degree oscillation.

In Fig. 4, the elliptical star images associated with the “Optical tagging method” for denoting which FOV the stars were imaged, are shown. Notice that astigmatism distortion is deliberately introduced (reference [2]) to cause the normally circular PSF to become elliptical. The eigenvalue of the figure inertia tensor associated with the “stretched” direction (eigenvector) indicates the FOV of origin for each star image. For orthogonal star fields and the adopted optical tagging method [2], the major axes of the stretched PSFs are nominally about 90 degrees apart. For example, in Fig. 4, the vertically stretched elliptical images are from the left FOV,

FIG. 2. Two Superimposed Images for One FOV.
Predictive Centroiding for Star Trackers with the Effect of Image Smear

The Star Coordinates for → ω = 0.030056 0.01995 0.15205

<table>
<thead>
<tr>
<th></th>
<th>stars at time t</th>
<th>stars at time t+5 t</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
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<tr>
<td>0</td>
<td></td>
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</tr>
<tr>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


FIG. 4. Two Superimposed Images for Two FOVs.

whereas the horizontally stretched images are from the right FOV. Obviously, the shape of the images can easily be detected during image processing by using the eigenvalue ratio between the inertia principal axes of the mask surrounding each
noncircular star image. Alternatively, we can simply compute the second moments in lieu of finding the eigenvalues of the covariance matrices. For noncircular PSF, this approximation becomes essentially exact. From a mathematical point of view, if \([x_i, y_i]\) are coordinates of the centroid, then

\[ J_1 = \sum_{i} (y_i - \bar{y})^2 \quad \text{and} \quad J_2 = \sum_{i} (x_i - \bar{x})^2 \]  

are the second moments, where the sums are extended to all of the significantly illuminated pixels (coordinates \([x_i, y_i]\)) used for centroiding. Therefore, the FOV identification is simply dictated by \(J_1 < J_2\) or \(J_1 > J_2\), provided that \(|J_1 - J_2|\) is greater than a given numerical threshold. Typically \(|J_1 - J_2|/J_1 > 0.1\) has been found reliable.

The number of pixels processed is reduced from \(512 \times 512 = 262,144\) to approximately \((n \times m)\), where \(n\) is the number of stars and \(m\) is the number of pixels associated with the average star images. For a typical image, \(n \times m = 8 \times 25 = 200\), so the amount of image data involved is four orders of magnitude less. So, because we process fewer numbers of pixels each time, these results show that the time required to find the centroids using predictive centroiding is less by over one order of magnitude than the time required using the regular centroiding techniques [3]. The computational overhead associated with the predictive centroiding calculations partially offsets the large reduction in pixel data, so that only one order of magnitude of computational savings is possible with our current implementation.
The Effect of Image Smear

Image smear occurs in the focal plane of the imaged stars for slow values of the integration time $t_s$, or for high values of the spacecraft angular velocity $\omega$. Interestingly, the above algorithms for the centroiding elliptical PSFs are applicable, to a degree, to the image smear problem. Figure 6 shows a night sky star image for a randomly oriented start tracker with a $657 \times 495$ pixel focal plane detector and 55 mm focal length ST-237 camera; the exposure time ($t_e$) for this image is 1.0 second. A high precision telescope mount is used to perform an angular rotation of the camera around a certain axis of rotation. Figure 7 shows the same star image as in Fig. 6 with smear $\omega = 1/12$ deg/sec and for $t_s = 0.5$ sec. Figure 8 shows the star image with image smear $\omega = 0.5$ deg/sec and for $t_s = 0.5$ sec. Figure 9 shows the star image with image smear $\omega = 0.5$ deg/sec and for $t_s = 2.0$ sec. These night sky images validated the fact that the volume of the intensity distribution of each star will be approximately constant during the integration time while the significantly illuminated area will increase while increasing the smear. These qualitative remarks break down at higher slew rates, when the accumulated energy becomes comparable to pixel dark current noise. We note that implementing the split field of view, with astigmatism tagging, presents a difficulty if there is significant image smear because it will be difficult to distinguish between smear and astigmatism. As a consequence, the astigmatism idea will restrict the angular velocity to be below a threshold value with negligible image smear. But more generally, some of the image processing ideas for astigmatic elliptical star images can be used for smeared images.

FIG. 6. Star Image Without Smear at 1.0 sec Exposure Time.
FIG. 7. Star Image at 0.5 sec Exposure and Smear = 1/12 deg/sec.

FIG. 8. Star Image at 0.5 sec Exposure and Smear = 0.5 deg/sec.
FIG. 9. Star Image at 2.0 sec Exposure and Smear = 0.5 deg/sec.

Maximum Angular Rate Estimation

The centroiding techniques allow the star direction to be determined with a precision of 1/10 of a pixel or better; this conservative estimate is derived from night sky experiments. From Fig. 10 we can calculate the corresponding maximum allowed separation angle for totally negligible image smear as \( \Delta \vartheta = 1/10 \) (chip size/number of pixels). The critical condition \( \Delta \vartheta = \omega \delta t \), where \( \delta t \geq t \), is the time between successive images and \( \omega \) is the angular rate of the spacecraft, corresponds to image smear \( \Delta \vartheta \) being equal to the random, approximately Gaussian centroiding errors. The analytical values of the maximum angular velocity for negligible image smear as a function of integration time are calculated for a typical 512 \( \times \) 512 CCD camera and a 7° field of view. Table 1 summarizes the results of the analytical value of the maximum angular velocity for each integration time.

For \( \omega < \omega_{\text{max}} = \Delta \vartheta / \delta t \) results in negligible image smear (smaller than expected centroiding error), however, we find in practice that acceptable centroid accuracy (50 \( \mu \) rad) may be obtained, for perhaps two or three times these “max” angular velocity estimates; this is discussed in the next section.

Measurement Errors and the Standard Deviation Computation

For the star images taken with smear effect, the attitude associated with the image should be most value at \( (t + \delta t/2) \). For simplicity, in the present discussion we consider \( t = \delta t \). So, without smear the \( \Delta \vartheta = \omega \delta t \) is negligible and the attitude matrix is \( C(t) \). For the smear case the attitude matrix at \( (t + \delta t/2) \) is

\[
C(t + \delta t/2) = [I - \bar{\omega} \delta t/2]C(t)
\]

(5)
TABLE 1. Analytical Maximum Angular Velocity

<table>
<thead>
<tr>
<th>( t ) (msec)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{\text{max}} ) (deg/sec)</td>
<td>0.156</td>
<td>0.078</td>
<td>0.052</td>
<td>0.0313</td>
</tr>
</tbody>
</table>

The equivalent angular centroiding (measurement) error is estimated from a finite sample \( N \) as

\[
\sigma^2 = \frac{1}{N} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\theta_{ij}^\text{smeared} - \theta_{ij}^\text{true})^2
\]

(6)

where \( \theta_{ij}^\text{smeared} \) is the interstar angle measured from the smeared image, \( \theta_{ij}^\text{true} \) is the corresponding interstar angle from catalogued vectors, \( N = n(n - 1)/2 \) is the number of star pairs, and \( n \) is the number of measured stars. The night sky images for the star images with and without smear are used to calculate the measurement errors, for different angular velocities and integration times. Notice \( \sigma^2 \) can be computed from measured and cataloged interstar angles without using an attitude estimate.

Table 2 shows the effect of the image smear on the centroiding errors for integration times equal to 10 to 50 msec, respectively. This table is created using a finite number of night sky images, but is believed to be converged to within 1% or better. Notice that accuracy degrades slowly with \( \omega \), until \( \omega = 2\omega_{\text{max}} \); see Table 1.

TABLE 2. Measurement Errors for Star Images for Various Cases

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>( \omega ) (deg/sec)</th>
<th>0.01</th>
<th>0.02</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( \sigma_{\text{true}} ) (\mu rad)</td>
<td>34.0</td>
<td>39.8</td>
<td>42.4</td>
<td>48.9</td>
<td>58.3</td>
<td>78.6</td>
<td>115.0</td>
</tr>
<tr>
<td>50</td>
<td>( \sigma_{\text{true}} ) (\mu rad)</td>
<td>37.0</td>
<td>43.5</td>
<td>49.8</td>
<td>75.7</td>
<td>262.4</td>
<td>456.3</td>
<td>657.4</td>
</tr>
</tbody>
</table>
As expected the measurement errors get worse with increasing smear. So, by comparing Table 1 with Table 2 we can conclude that the actual angular velocity can be increased by factor of about two to three from the analytical maximum angular velocity estimate in Table 1, with small to moderate accuracy degradation.

Conclusion

This paper presents predictive centroiding as a new approach for fast image processing for star trackers with split FOV. It enables only several hundred pixels to be processed (as opposed to $10^3$ or $10^6$ pixels); this approach is especially well-suited to Active Pixel cameras which permit random access of the pixel response due to selected stars. The speed and the accuracy of this approach is successfully demonstrated in comparison with the ordinary centroiding algorithms which do not use the previous image data. The predictive centroiding algorithm will be used in the GIFTs EO-3 mission in 2004. Also, the image smear problem is studied for the case of high spacecraft angular velocity, or long integration times for the camera. Similar algorithms can be used for centroiding the noncircular point spread functions. Image smear can present significant problems, however, for the astigmatism based optical tagging in split field of view cameras because the astigmatic PSFs are further distorted by smearing. This problem requires further study. Alternative methods for optically tagging the stars for the two fields of view are being evaluated. A critical angular velocity is introduced, below which smearing is negligible. For smearing non-astigmatic PSFs, simulation results indicate the actual spacecraft angular velocity can be increased to two or three times the value of the critical analytical angular velocity estimate and the centroid measurement errors will remain within allowable limits.

References