Recursive On-orbit Calibration of Star Sensors

D. Todd Griffith\textsuperscript{1}
John L. Junkins\textsuperscript{2}

\textbf{Abstract}

Estimation of calibration parameters for a star tracker is investigated. Conventional estimation schemes are evaluated for the solution of the time varying calibration parameter problem. These results show that standard estimation schemes can be utilized to solve the calibration problem on-orbit. Proper tuning of the time dependent estimation schemes shows that the calibration parameters can be estimated well even when they vary over the course of one orbital period.

\textbf{Introduction}

Star trackers are primarily used to determine a spacecraft’s attitude. One of the most important issues regarding precise attitude determination with star trackers is calibration. It is desired that calibration account for any mechanism which tends to distort, usually relative to the pin hole camera ideal model, the imaged data. These distortion mechanisms can include those related to the sensor including non-planarity and instrument aging and those related to the optics including lens distortion and misalignment.

Two approaches have been developed for estimating the image distortion\textsuperscript{1}. In the attitude dependent approach, the estimated attitude matrix is used to estimate the line of sight measurements in the body frame. This estimate can then be compared to the measured line of sight vectors in order to quantify the sensor precision. In the second approach, the attitude independent approach, the interstar angles are utilized to quantify sensor precision.

The Recursive Least Squares estimation approach is useful for solving the nominal stationary calibration problem. In addition this approach is useful for the slowly time varying calibration problem when a sufficiently large star data rate is available. The time varying distortion mechanisms can include instrument aging and temperature effects. Instrument aging effects will occur over the course of several months or years, and fall into the category of the slowly time varying calibration problem. In this case the calibration parameters are changing slow enough so that on-orbit health monitoring should sense what changes are taking place in the system, and can act to improve performance. However, temperature effects should be expected to effect system performance more quickly, that is, at orbital rate.

The attitude dependent and attitude independent calibration approaches have been evaluated by simulation\textsuperscript{1}. Future work involves evaluating these calibration approaches with experimental testing. Ground-based testing can and has been used for an initial

\textsuperscript{1} Graduate Assistant Research, Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, griffith@tamu.edu, AIAA member.

\textsuperscript{2} George Eppright Chair, Distinguished Professor, Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, junkins@tamu.edu, AIAA fellow.
calibration for star trackers prior to launch. However for the purpose of algorithm validation, ground based testing is a tool well suited for characterization of the size and the functional form of the distortion. In addition, ground based testing is very useful in that extensive experiments can be done independent of knowledge of the sensor orientation. These studies are designed to establish and validate an autonomous on-orbit calibration algorithm. Orbital conditions cannot be exactly achieved on the ground; however, the most prominent conditions of the orbital environment can be simulated individually. These simulation studies provide a basis for forthcoming ground-based testing.

**Star Tracker Mission Description**

The EO-3 GIFTS mission is planned to have a duration of several years. It should be expected that the properties of the camera, or equivalently the calibration parameters, will vary over time. Then, an important question is “How quickly will the calibration parameters vary?” If the calibration parameter changes are small over the course of several weeks or months, then the requirement on calibration update rate will not need to be as stringent, and the complexity of the required estimation algorithm will not be as strict. However, if the calibration parameters vary significantly over the course of a single orbital period, then this must be accounted for by taking an approach that not only incorporates health monitoring but is more flexible. One significant source for non-stationary could be temperature effects, which are likely to show up as the spacecraft incident temperature changes over the course of the 24-hour orbital period. The highest frequency disturbance to calibration parameter stationarity is the most important effect which should be accommodated in the algorithm. Therefore, in this paper we simulate and study the case where calibration parameters vary over the course of an orbital period proportional to changes in incident temperature.

**Recursive Estimation of Time-Varying Parameters**

The recursive Least Squares Algorithm has been shown to be capable of estimating the nominal stationary calibration parameters. This approach has the advantage that a large amount of information does not need to be stored. The calibration parameters are updated and the covariance is propagated each time a new image is available. Once the update is performed, the image data can be discarded. It is well known that this algorithm can result in what is known as covariance wind-up. That is, once an arbitrarily large number of updates have been performed, not only the estimates but the covariance converges. This is a limitation since new data cannot significantly impact, at least not in a timely fashion, the estimates for the calibration parameters. This indicates that the algorithm thinks the solution has been reached.

The covariance wind-up problem can be approached in several ways. One solution is to simply reset the covariance matrix occasionally so that the estimator does not become too overconfident. Another solution is to utilize estimation schemes with additional tuning parameters. Qualitatively, these tuning parameters should allow the estimate to more accurately reflect the new information. This would alleviate the major limitation of the nominal algorithm. In addition, these tuning parameters should be easy to implement such that specific requirements can be met. For example, if the to be estimated parameters vary at a certain known rate, then the known rate should be
informative as to a proper choice for tuning parameters. Two such algorithms, which are extensions of the nominal algorithm, are available for this application. These algorithms include additional tuning parameters which can be used to accommodate estimation of time varying parameters.

In the following sections, we merely summarize the key points and main results for each of the estimation schemes utilized in this paper.

**Recursive Least Squares**

For the standard recursive Least Squares problem we seek to minimize the following cost function with prediction error given by Eqn. 2:

\[
J = \frac{1}{2} e^T W e = \frac{1}{2} (\tilde{y} - Ax)^T W (\tilde{y} - Ax) \tag{1}
\]

\[
e = \tilde{y} - Ax \tag{2}
\]

Given this standard definition for the cost to be minimized, the coefficient update and covariance propagation equations for the standard Recursive Least Squares algorithm can then be summarized as follows:

\[
\hat{x}_{k+1} = \hat{x}_k + P_{k+1} A^T_{k+1} W_{k+1} \{\tilde{y}_{k+1} - A_{k+1} \hat{x}_k\} \tag{3}
\]

\[
P_{k+1} = P_k - P_k A_{k+1} \left(W_{k+1}^{-1} + A_{k+1} P_k A_{k+1}^T\right)^{-1} A_{k+1} P_k \tag{4}
\]

Here, \(A\) is a vector of chosen basis functions, \(\hat{x}\) is the to be estimated set of coefficients corresponding to \(A\), \(P\) is the covariance matrix, and \(\tilde{y}\) is the measurement vector. \(W\) is a weighting matrix, which should be thought of as a tuning parameter. The correct weighting matrix for a particular problem can be chosen from several points of view. The prevalent idea is to keep in mind that those measurements which are, in some sense, more important are given more weight (larger value). Therefore, a few choices for the weight matrix can be summarized as follows: 1) arbitrarily chosen, based on experience with the specific problem, 2) from a probabilistic standpoint, for uncorrelated measurements and stationary parameters, the optimal choice for \(W\) is \(W = \text{diag}(1/\sigma_i^2)\) where \(\sigma_i^2\) is the variance of the \(i\)-th measurement, and 3) embedded with a forget factor. Typically, the first choice has more utility in the batch Least Squares estimation, not for real time estimation. The second choice is useful for both batch and recursive estimation, and is commonly referred to as minimum variance estimation. The primary utility of the third choice is to improve the estimates when the parameters are non-stationary. As is indicated below, the most recent data is given the highest weight.

**Recursive Least Squares with Covariance Update Modification**

One modification that can be made to the nominal algorithm given in eqns. (3) and (4) is to simply add a covariance-like (symmetric positive definite) matrix to the right hand side of the covariance update equation. This approach can be justified by considering that the model for the propagation of the states (the to be estimated...
calibration parameters) includes an additive zero-mean Gaussian noise term. Intuitively, this approach is appealing since we can recognize that if the covariance matrix (if computed by Eqn. 4) were to get very small, then adding a covariance-like matrix to the update will, in essence, set a lower bound on the propagated covariance. We can choose this additive matrix as a tuning parameter to ensure that the estimator does not become overconfident in the estimate. Hence, the coefficient update equation remains the same and covariance propagation equation takes on a new form as given in Eqn. 6.

\[
\begin{align*}
\hat{x}_{k+1} &= \hat{x}_k + P_{k+1} \hat{A}^T_{k+1} W_{k+1} \left\{ \tilde{y}_{k+1} - A_{k+1}\hat{x}_k \right\} \\
P_{k+1} &= P_k - P_k A_{k+1}^T \left( W_{k+1}^{-1} + A_{k+1} P_k A_{k+1}^T \right)^{-1} A_{k+1} P_k + Q_k
\end{align*}
\]

(5) \hspace{1cm} (6)

The new form of Eqn. 6 includes the additive term \( Q_k \). This matrix is a constant, and ideally is given by \( E[w_k w_j^T] = \begin{cases} 
0 & k \neq j \\
Q_k & k = j
\end{cases} \), where the state errors are not correlated forward or backward in time.

**Recursive Least Squares with Forget Factor**

We can also choose a different structure for the weight matrix \( W \) given in Eqn. 1. As opposed to choosing \( W \) from a statistical or probabilistic point of view, we can choose a form which favors the most recent measurements:

\[
W = \begin{bmatrix}
\lambda^{n-1} & 0 & 0 & \cdots & 0 \\
0 & \lambda^{n-2} & 0 & \cdots & 0 \\
0 & 0 & \lambda^{n-3} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \lambda
\end{bmatrix}
\]

(7)

Here, the parameter \( \lambda \) is the so-called forget factor, which can take values of \( 0 < \lambda \leq 1 \), and \( n \) is the number of measurements. Note that for \( \lambda < 1 \), the largest weight is given to the most recent measurement. When \( \lambda = 1 \), the forthcoming equations reduce to the standard recursive Least Squares solution given by Eqns. 3 and 4. This is also referred to as the infinite memory version of the algorithm. Substituting Eqn. 7 into Eqn. 1, we begin the derivation of the following update equations.

\[
\begin{align*}
\hat{x}_{k+1} &= \hat{x}_k + P_{k+1} \hat{A}_{k+1} \left( \lambda + \hat{A}_{k+1}^T P_{k+1} A_{k+1} \right)^{-1} \left\{ \tilde{y}_{k+1} - A_{k+1}\hat{x}_k \right\} \\
P_{k+1} &= \lambda^{-1} \left( P_k - P_k A_{k+1} \left( \lambda + A_{k+1}^T P_{k+1} A_{k+1} \right)^{-1} A_{k+1}^T P_k \right)
\end{align*}
\]

(8) \hspace{1cm} (9)

Note that it is assumed that we operate on only one measurement at a time; therefore, \( \lambda \) is a scalar and each of the inversions in Eqns. 8 and 9 can be computed by simple division.
In summary, for each of these algorithms we can initialize them with a starting guess for the to be estimated coefficients, covariance, and additional tuning parameters. A suitable starting guess for the coefficients is a zero vector. In addition, covariance tells us something important about how well we know the truth. Therefore, a good choice for the initial covariance is a matrix with large values on the diagonal. With this choice, we begin the process by assuming that we don’t have much confidence in our initial guess. On the other hand, the tuning parameters should be chosen with more caution since no provisions have been made for their adaptation.

In the following sections, we evaluate each of these algorithms as a solution for the on-orbit calibration problem. But first, we will describe the forthcoming simulations.

**Simulated Spacecraft Maneuver Description**

As mentioned earlier, the GIFTS mission will operate in a geosynchronous orbit. Thus, a 24 hour simulation will encompass one entire orbital period. In order to make the orbital maneuver more realistic, small rate motion is superimposed on the GEO motion. When the rates are known, it is then possible to solve for the spacecraft attitude (quaternions) by closed form solution. These quaternions form the basis for the truth which underlies the simulated star data. That is, when the true quaternions are known, it becomes possible to then access the star catalog to obtain the inertial coordinates for the viewable stars. The known attitude can then be used to project or map these coordinates into the body (sensor) frame of the camera. These coordinates are the simulated star centroids, the true measurements. A known distortion function along with noise (17 micro-radian standard deviation) can then be added to the true measurements to form a set of observations which are intended to be the real simulated star coordinate data. For these simulations, we assume to have star data at a rate of 1 Hz. We anticipate 10 Hz or better for the GIFTS mission; however, experience shows that 1 Hz star data is sufficient for the purpose of this study.

A plot of the rate data and quaternions for this maneuver are given in Figs. 1 and 2, respectively. The rates of Fig. 1 were specified by superimposing 2 degree sinusoidal oscillations of the boresight axis at 0.05 degree per second. The true quaternions of Fig. 2 underlying the simulation were computed by closed form solution given the specified rates. Figs. 2-5 show the population of sensor with star measurements. When the boresight of the camera has some small oscillation, the measured stars will not follow straight-line trajectories across the sensor. This is advantageous in that the sensor frame is more quickly populated across the entire sensor, and the estimation scheme provides better results that are representative of the entire focal plane. As seen in Fig. 5, 24 hours of star data sufficiently covers the entire sensor frame. Note that one region near the center of the sensor frame contains no stars. This is not a desired result; however, it can be corrected by opening up the star catalog to look for stars in that region.
Figure 1. Rates for simulated maneuver

Figure 2. Quaternions for simulated maneuver

Figure 3. Star data after 3 hours

Figure 4. Star data after 12 hours

Figure 5. Star data after 24 hours
Distortion Parameter Estimation Results

The following results are generated for the simulated maneuver and star data shown in the previous section. Here, we will summarize the results for each of the algorithms introduced earlier. For each case, the observation equations are given by

\[
\Delta v = \begin{bmatrix}
-\hat{v}_z \\
\hat{v}_z \\
-\hat{v}_y + \hat{v}_x 
\end{bmatrix} = \begin{bmatrix}
-\hat{v}_z \Phi^T a \\
\hat{v}_z \Phi^T b \\
-\hat{v}_y \Phi^T b + \hat{v}_x \Phi^T a
\end{bmatrix}
\]

where \( \Delta v \) is the computed error \( \Delta v = \hat{v} - \hat{v} = \hat{v} - \hat{Cn} \). The tilde indicates that the quantity is a measurement, and the hat indicates that the quantity is an estimate. For the calibration problem, \( \Delta v \) is the measurement. Additional quantities of interest are defined as follows:

\[
\Phi^T = \begin{bmatrix}
v_x \\
v_y \\
v_z
\end{bmatrix}
\]

\[
a = \begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
a_6
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4 \\
b_5 \\
b_6
\end{bmatrix}
\]

The estimated star vector in the body frame is determined by the mapping of the catalogued star vectors (in inertial frame) into the body frame via the estimated attitude matrix. However, for these results the true known attitude matrix is used instead of the estimated attitude matrix. In this study, we are primarily interested in the convergence properties of the estimation schemes outlined earlier. The problem of star tracker calibration when the estimated attitude matrix is used to compute the error (when the estimated attitude also is corrupted by the biased measurements) has been studied\(^1\), but is not addressed in this paper.

In the following simulations, the nominal stationary true coefficients underlying the simulation are scaled by a sinusoidal function of the form, \( 1 + 0.1 \sin(\omega t) \). As such, 10% errors are introduced in the nominal coefficients. The parameter, \( \omega \), is chosen such that variation occurs identically over one complete orbital period (24 hours).

Results for Recursive Least Squares (no alteration)

The following results are for the standard Recursive Least Squares algorithm of Eqns. 3 and 4 with no additional tuning parameters or covariance reset.
Figure 6. Post calibration X-direction statistics

Figure 7. Post calibration Y-direction statistics

Figure 8. Post calibration residual correlation

Figure 9. Tracking of $a$ coefficients

Figure 10. Tracking of $b$ coefficients
These results show that when the standard recursive Least Squares algorithm is used, the estimated coefficients do not track well with the true coefficients as shown in Figs. 9 and 10. Figs. 6-8 show that, in particular, the coefficients are not tracked well beyond the first few hours. This performance is primarily because the covariance has converged rather quickly and the effect of new measurements is ignored. These results indicate that if the calibration parameters are varying with time, then some provision must be made in the algorithm to account the changes.

**Results for Recursive Least Squares with Covariance Update Modification**

The results of this section are for the algorithm described by Eqns. 5 and 6. It has been suggested the additional tuning parameter with this approach should be chosen based on the noise (amplitude of variation) of the states. It was found that when this approach was taken, the results are indistinguishable from those of the standard recursive Least Squares algorithm given in the preceding section. This is so because the matrix $Q_k$ has diagonal elements on the order of $10^{-12}$, and the resulting covariance modification is insignificant. Therefore, a suitable matrix $Q_k$ was chosen by trial and error, \[ \text{diag}(Q_k) = 10^{-4} \{0.05, 0.1, 0.1, 50, 1, 10\}. \] The following results are for those with this choice for $Q_k$.

These results show that tracking of the true coefficients is greatly improved with this choice of $Q_k$. Naturally, this choice for $Q_k$ is not the optimum one; however, here we’ve shown that this simple modification to the standard algorithm results in significant improvement in the performance of the calibration algorithm.

**Results for Recursive Least Squares with Forget Factor**

The results of this section are for the algorithm described by Eqns. 8 and 9. Here, the additional tuning parameter should be chosen based on what is called the memory length of the estimator, $L$. The relationship between the memory length and the parameter $\lambda$ is

![Figure 11. Tracking of a coefficients](image1)
![Figure 12. Tracking of b coefficients](image2)
\[
\lambda = e^{-\frac{1}{L}}
\]  

(12)

If we choose \( L = 10,000 \) samples (at 1 Hz = 10,000 seconds \(~2.8\) hours), then \( \lambda = 0.9999 \). The results for this choice of \( \lambda \) are given in Figs. 13-17.

Figure 13. Post calibration X-direction statistics  

Figure 14. Post calibration Y-direction statistics  

Figure 15. Post calibration residual correlation
With this approach, the results are quite similar to that of the previous section. Tracking performance is also quite good. In addition, it should be noted that the estimates of the constant offset coefficient are less noisy than that of the previous section. Progressing along the coefficient vector, it can be seen that the coefficient estimates of Figs. 16 and 17 become more noisy.

It is anticipated that a local approximation scheme\textsuperscript{7.1} will be utilized in this calibration effort. For this regime, the approach using a forget factor would provide the most straightforward implementation since only a single parameter would need to be specified. This is advantageous since an individual tuning parameter will need to be specified for each local approximation. When this tuning parameter is a single scalar, the complexity of the problem is reduced. As can be seen with the approach with covariance modification, 6 additional tuning parameters need to be specified. Another important advantage of utilizing a forget factor is that when adaptation of the tuning parameters is desired, a more appropriate value for the forget factor can be found by the exponential relation given in Eqn. 12. One would then need to determine what memory length, L, is necessary to provide good tracking performance.

Conclusions

Results for estimation of time varying calibration parameters have been presented. Two approaches have been shown to be capable of estimating the time varying calibration parameters in real-time. Future work will include analysis of experimental data taken with prototype star cameras. This exercise will provide insight into the stationarity of the calibration parameters, and in turn the applicability of one of the two modified recursive Least Squares estimation schemes to the solution of the on-orbit calibration problem. Flexibility in the calibration algorithm is very important since this system will be operating autonomously. Any tuning of the calibration algorithm will be done on-orbit which requires a set of rules to be in place that dictate how the calibration algorithm adapts to any departure from stationarity of the calibration parameters.
References


