VISION BASED SENSOR AND NAVIGATION SYSTEM
FOR AUTONOMOUS AERIAL REFUELING

John Valasek*, Kiran Gunnam†, Jennifer Kimmett‡, and John L. Junkins**
Texas A&M University, 3141 TAMU, College Station, TX 77843-3141
and
Declan Hughes‡
StarVision Technologies Inc., College Station, TX 77840

Abstract
Autonomous in-flight aerial refueling is an important capability for the future deployment of unmanned aerial vehicles, since they will likely be ferried in flight to overseas theaters of operation instead of being shipped unassembled in containers. A reliable sensor, capable of providing accurate relative position measurements of sufficient bandwidth, is key to such a capability. This paper introduces an innovative vision based sensor and navigation system that enables precise and reliable probe-and-drogue autonomous aerial refueling for unmanned aerial vehicles. A performance robust controller is developed and integrated with the sensor system,

*Associate Professor and Director, Flight Simulation Laboratory, Aerospace Engineering Department. Associate Fellow AIAA. valasek@aero.tamu.edu
†Graduate Research Assistant, Aerospace Engineering Department.
**Distinguished Professor, George J. Eppright Chair, and Director, Center for Mechanics and Control, Aerospace Engineering Department. Fellow AIAA. junkins@tamu.edu
‡Chief Engineer. dhughes@starvisiontech.com

Department of Aerospace Engineering
Texas A&M University
3141 TAMU
College Station, TX 77843-3141
and feasibility of the total system is demonstrated by simulated docking maneuvers with a stationary probe subjected to turbulence. A Vertical/Short Takeoff & Landing unmanned air vehicle model based on a scaled-down AV-8B is used for controller design and simulation. Results indicate that the integrated sensor and controller enables precise aerial refueling, including consideration of realistic measurement errors and disturbances.

Introduction

There are currently two approaches in use for aerial refueling. The United States Air Force uses the flying boom developed by Boeing. The boom approach is supervised and controlled by a human operator from an operator station near the rear of the tanker aircraft. The boom operator is responsible for “flying” the boom into the refueling port on the receiver aircraft. In this method, the job of the receiver aircraft is to maintain the proper refueling position with respect to the tanker, but leave the fine structure aspects of the control to the human operator in the tanker. The probe-and-drogue refueling system is the standard for the United States Navy and the air forces of most other nations. In this method, the tanker trails a hose with a flexible “basket”, called a drogue, at the end. The drogue is aerodynamically stabilized. It is the responsibility of the pilot of the receiver aircraft to maneuver the receiver’s probe into the drogue (Figures 1 & 2). This is the preferred method for small, agile aircraft such as fighters because both the hose and drogue are flexible and essentially passive during re-fueling; a human operator is not required on the tanker.\textsuperscript{1,2,3}

Autonomous in-flight refueling using a probe-and-drogue system is basically a docking situation that requires approximately 0.5 cm accuracy in the relative position of the refueling probe (from the receiving aircraft) with respect to the drogue (from the tanker) during the end-game. Currently, autonomously refueling of aircraft in-flight using the probe-and-drogue
method has not been accomplished. The maturation of the technology requires several issues to be address, with the most fundamental being the lack of sufficiently accurate/reliable relative motion sensors. Some methods that have been considered for determining relative position in a refueling scenario include the Global Positioning System (GPS), and visual servoing with pattern recognition software. GPS measurements have been made with 1 cm to 2 cm accuracy for formation flying, but this is not accurate enough for in-flight refueling, as problems associated with lock-on, integer ambiguity, and low bandwidth are additional disadvantages. Pattern recognition codes are not sufficiently reliable in all lighting conditions, and with adequate fault tolerance, may require extreme amounts of computational power in order to converge with sufficient confidence to a solution.

In the late 1990s, Texas A&M researchers began developing an innovative Vision-based Navigation system called VisNav, that is capable of accurately providing the needed six degree-of-freedom information for real-time navigation with high precision. VisNav will enable accurate autonomous aerial refueling without extensive alterations in the current refueling systems. VisNav is a cooperative vision technology whereby the beacons mounted on a target body (e.g., the drogue) are supervised by the VisNav sensor on a second body (e.g., the receiver aircraft). The drogues, for several years, have been conventionally equipped with Light Emitting Diodes (LED) to aid human pilots with night refueling operations, and VisNav can use either the existing LEDs or ideally infra-red LEDs as beacons. The only major equipment change to the probe-and-drogue legacy system is the addition of the VisNav sensor located on the UAV itself and beacon controllers for the structured light LEDs on the drogue.

This paper demonstrates the capability of the VisNav system to accurately determine the position and attitude of the UAV in relation to a stationary drogue. After detailing the VisNav system, linear models of a UAV are used to first design the robust controller using the
proportional-integral-filter optimal nonzero setpoint (PIF-NZSP) with control rate weighting, and sensor models of the VisNav system are integrated with the controller and evaluated on a six degree-of-freedom UAV simulation. The Dryden gust model with moderate turbulence is used to assess disturbance rejection characteristics in the presence of exogenous inputs.

**Vision Based Navigation Sensor and System**

The innovative VisNav sensor comprises a new kind of optical sensor combined with structured active light sources (beacons) to achieve a selective or “intelligent” vision. VisNav structures the light in the frequency domain, analogous to radar, so that discrimination and target identification are near-trivial even in a noisy ambient environment. This is accomplished by fixing several LED based beacons to the target frame A, and an optical sensor, based on a Position Sensing Diode (PSD), to the aircraft frame B. The LEDs emit structured light modulated with a known waveform; filtering of the received energy then allows most of the much larger ambient energy to be rejected. In addition, the power of each LED is adaptively adjusted to optimize the received energy amplitude to maximize the signal to noise ration of each individual measurement; all of this can be routinely accomplished in a wireless feedback loop closed at 100HZ.

When light energy from an individual beacon is focused on the surface of the PSD, it generates four electrical currents with four pickoff leads, one on each side (Figure 3). The closer the image centroid is to a given pickoff lead the stronger the current through that lead. From the current imbalance in the two horizontal leads the horizontal displacement of the light centroid can be estimated. Similarly, the current imbalance in the vertical leads provides an estimate of the vertical displacement of the image centroid. In reality, there is weak cross channel coupling and other nonlinearities.
A nonlinear mapping is determined in a one-time calibration to linearize this relationship; the calibration functions are applied in real time. These horizontal and vertical image centroid displacement measurements vary in a monotonic relationship with the azimuth and elevation of the beacon source with respect to the local sensor frame. When this data is collected for four or more separate beacons, a Gaussian Least Squares Differential Correction (GLSDC) routine is used to calculate the six degree-of-freedom sensor position and attitude data with respect to the target frame A at an update rate of 100 Hz.\textsuperscript{10-12}

The basic architecture of the VisNav system is shown in Figure 4. The sensor digital signal processor (DSP), on the sensor aircraft, selects a beacon from the available set, and transmits the beacon number and desired intensity via an IR digital optical link to the beacon controller (BC) on the tanker aircraft (actually, on the drogue). The beacon controller responds by turning on the selected beacon at the requested intensity. The 2D position sensing diode (PSD, a type of photodiode, lower right-hand corner of the sensor block) generates four electrical currents, left (L), right (R), up (U) and down (D), in response to the IR light image of this beacon. The four currents contain a carrier frequency to provide discrimination against sunlight, engine energy, etc., and must be amplified and demodulated before conversion to digital form for the DSP CPU. After the DSP has selected at least four distinct beacons in sequence, and the four PSD signals have been collected for each, the algorithm can compute the 6DOF position/attitude of the sensor aircraft with respect to the BC aircraft, in the BC aircraft frame of reference, and vice-versa. A significant feature of the PSD pre-amplifier is its ability to operate in the presence of sunlight orders of magnitudes larger than the beacon signal, close to the associated shot-noise limit.

The VisNav system can be used in a wide variety of lighting conditions due to the robustness gained through the use of frequency structured light and subsequent filtering in the sensor. The beacons emit light modulated at 40 KHz and the sensor filters out all light excepting that which
lies close to 40 KHz. This frequency was chosen as a compromise between avoiding lower frequency energy from natural and artificial light sources, and avoiding increased sensor amplifier noise effects at higher frequencies. Furthermore, a colored glass optical filter is placed outside the sensor lens in order to block virtually all visible light (including about 95% of the solar energy) while passing the Infra-Red (IR) wavelengths produced by the beacons. This passive filter greatly reduces random shot noise, which is proportional to the square root of all energy incident on the detector. The near IR wavelength is chosen to lie close to the optimal response wavelength of the Silicon PSD.$^{10-12}$

If the power output from a particular beacon is too high, one or more of the PSD transimpedance amplifiers may saturate making determination of the light image centroid coordinates inaccurate. If the energy is too low, then a poor signal to noise ratio is achieved. To handle these situations feedback control is used to maintain a beacon light output intensity that produces an approximately 70% full scale response at the output of the sensor amplifiers. This is a one step algorithm where knowledge of the beacon intensity and sensor response level at the previous activation allows calculation of the required beacon intensity to bring the response to 70%, assuming no change in the relative positions of the beacons and sensor. In practice relative movement, slight unmeasured nonlinearities, and noise/disturbances prevent perfect compensation. The beacon intensity information is conveyed from the sensor to the beacons via an IR optical or radio signal.$^{10-12}$

The VisNav system is currently at the laboratory hardware stage, and several hardware prototypes have been built and successfully tested. Figure 5 shows a version 3.2 VisNav sensor box, a PSD sensor, and part of the pre-amplifiers. The version 4.0 VisNav sensor will have less than one third of the volume of the version 3.2 article shown; much smaller versions are practical, and are under development. The VisNav Laboratory website $^{14}$ contains additional
details and hardware descriptions. For the aerial refueling application, Figure 6 shows a conceptual installation of VisNav LED beacons on the drogue basket. The IR VisNav beacons would be attached at the same locations on the drogue currently occupied by visible LED navigation lights.

**Estimation of Relative Positions**

The normalized voltages computed from the PSD imbalance currents due to the $i^{th}$ target are mapped to the horizontal and vertical displacement $(y_i, z_i)$ estimates of that target beacon’s image spot with respect to the sensor frame. The mapping compensates for primarily lens induced distortion and may employ Chebyshev polynomials, or some other calibration functions to compensate for these distortions. The calibration process maps the measured voltages $(V_{y_i}, V_{z_i})$ into $(y_i, z_i)$ consistent with the known camera position, known object space beacon location, and the colinearity equations (pin-hole camera model). In other words, the calibration function compensates for all systematic non-ideal effects. Here $(V_{y_i}, V_{z_i})$ are the normalized voltages; $X_i, Y_i, Z_i$ are the known object space coordinates of the target light source ($i^{th}$ beacon); $X_c, Y_c, Z_c$ are the object space coordinates of the sensor origin attached to the lens center fixed in the vehicle; $C$ is the direction cosine matrix of the image space coordinate frame with respect to the object space coordinate frame; $F_y, F_z$ are the lens calibration maps; $f$ is the focal length of the sensor lens and $y_i, z_i$ are the ideal image spot centroid coordinates in the image space coordinate frame for the $i^{th}$ target light source.

The sensor electronics, the ambient light sources, and the inherent properties of the sensor produce noise in the PSD measurements. To a good approximation this noise can be modeled as zero mean Gaussian noise. The measurement model $h_i(x)$ is
\[ \dot{h}_i(x) = \tilde{h}_i(x) + v_i \]

where \( x = \begin{bmatrix} L \\ P \end{bmatrix} \) is the state vector of the sensor, and \( L = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} \) is the position vector, \( p = \begin{bmatrix} p^1 \\ p^2 \\ p^3 \end{bmatrix} \) is the orientation/attitude vector, expressed in terms of modified Rodrigues Parameters (MRP). The modified Rodrigues Parameters are defined as \( p = e \tan(\Phi / 4) \), where \( e = [e_1 \quad e_2 \quad e_3] \) is the principal rotation axis, and \( \Phi \) is the principal rotation angle. The vector \( \tilde{h}_i(x) \) is an ideal measurement model, and is a function of \( y_i(x) \) and \( z_i(x) \):

\[ \tilde{h}_i(x) = F(y_i(x), z_i(x)) \]

where \( v_i \) is the Gaussian measurement noise with covariance \( R_i = \mathbb{E}\{v_i, v_i^T\} \).

The use of a Gaussian Least Squares Differential Correction (GLSDC) algorithm to determine the states, attitude and position, gives a best geometric solution in the least square sense upon convergence through iterations. Estimates for position and attitude are refined through iterations of GLSDC as it minimizes the weighted sum of squares

\[ J = \frac{1}{2} (h - \tilde{h})^T W (h - \tilde{h}) \]  

(1)

where \( W \) is the weighting matrix and

\[ W_{i,j} = 1 / R_{i,j} = \mathbb{E}\{v_i, v_j^T\} \]  

(2)

From the geometric colinearity equations, ideal image centroid coordinates are given by

\[
y_i = g_{yi}(X_i, Y_i, Z_i, X_c, Y_c, Z_c, C) \\
= -f \left( \frac{C_{21}(X_i - X_c) + C_{22}(Y_i - Y_c) + C_{23}(Z_i - Z_c)}{C_{11}(X_i - X_c) + C_{12}(Y_i - Y_c) + C_{13}(Z_i - Z_c)} \right) \\
z_i = g_{zi}(X_i, Y_i, Z_i, X_c, Y_c, Z_c, C) \\
= -f \left( \frac{C_{31}(X_i - X_c) + C_{32}(Y_i - Y_c) + C_{33}(Z_i - Z_c)}{C_{11}(X_i - X_c) + C_{12}(Y_i - Y_c) + C_{13}(Z_i - Z_c)} \right)
\]
An alternate vector representation for the above equations can be written in the unit line of sight vector form\textsuperscript{13}

\[ \mathbf{b}_{ij} = \mathbf{C} \mathbf{r}_i \]

where \( \mathbf{b}_{ij} = (1/\sqrt{f^2+y_i^2+z_i^2})[\mathbf{f} \ y_i \ z_i]^T \) are the sensor frame unit vectors,

\[ \mathbf{r}_i = (1/d_i) \cdot [(X_i - X_e) \ (Y_i - Y_e) \ (X_i - X_e)]^T \]

are the object frame unit vectors, and

\[ d_i = \sqrt{(X_i - X_e)^2+(Y_i - Y_e)^2+(Z_i - Z_e)^2} \]

MRPs are derived from the Quaternions and yield better results in terms of linearity since they linearize like quarter-angles instead of half-angles for the Quaternions, the interested reader is referred to [13] for more details.

The direction cosine matrix in terms of modified Rodrigues parameters is

\[ C = I_{3 \times 3} + \frac{8(p \times)^2 - 4(1 - p^T p)(p \times)}{(1 + p^T p)^2}, \]

and

\[ p \times = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \]

**Measurement Model**

Since \( f \) is constant, there is no need to consider this parameter and redundancy is eliminated because the measurement sensitivity matrix \( H \) in (7) is \( 2N \times 6 \) instead of \( 3N \times 6 \), where \( N \) is the number of beacons and the weighting matrix \( W \) in (2) is \( 2N \times 2N \) instead of \( 3N \times 3N \). The measurement model used has only two parameters but normalizes them in order to further linearize the GLSDC model, and thus also improve convergence performance\textsuperscript{15}.
\[ h_i = (1/ \sqrt{f^2 + y_i^2 + z_i^2})[y_i, z_i]^T \]  \hfill (3)

This can be represented as

\[ h_i(x) = D \ r_i \]  \hfill (4)

where \( D_{j,k} = C_{j+1,k} \), \( j = 1,2; k = 1,2,3 \). The measurement sensitivity matrix for the \( i \)th beacon \( H_i \) is obtained by partial differentiation of the measurement model with respect to the state vector \( x \):

\[
H_i = \frac{\partial h_i}{\partial x} = \begin{bmatrix} \frac{\partial h_i}{\partial L} & \frac{\partial h_i}{\partial Q} \end{bmatrix}
\]

\[
\frac{\partial h_i}{\partial L} = -D \ {(I_{3 \times 3} - r_i r_i^T)} / d_i
\]

\[
\frac{\partial h_i}{\partial Q} = \frac{4}{(1 + p^T p)^2} \{ S \} \{(1 - p^T p)I_{3 \times 3} - 2 [p \times] + 2 p p^T \}
\]

with \( S = \begin{bmatrix} s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}, \ s = [s_1 \ s_2 \ s_3]^T = C \ r_i \). The actual measurement matrix for \( N \) beacons using (3) is

\[ b = [h_i^T \ ... \ h_N^T]^T \]  \hfill (5)

and the estimated ideal measurement matrix using estimated position and orientation in the colinearity equation (4) is

\[ \tilde{b} = [\tilde{h}_1^T \ ... \ \tilde{h}_N^T]^T \]  \hfill (6)

The measurement sensitivity matrix is

\[ H = [H_1^T; H_2^T; ...; H_N^T]^T \]  \hfill (7)
We mention that a sensor calibration is required to account for lens distortion, detection nonlinearity, and departures of the actual sensor behavior from the ideal pin-hole camera model implicit in the formulas above. This calibration process operates on the sensor output to map the measurements to offset values that are adequately modeled by the co-linearity equations.

GLSDC Algorithm

The algorithm operates as follows. The initial state estimate before iteration is $\hat{x}_{k,0} = \hat{x}_{k-1}$. If $k = 0$ we use the estimate $\hat{x}_{k,0}$. Iterations are then performed using the standard Gaussian Least Squares Differential Correction (GLSDC) procedure given by (8), where $P_{k,i}$ is the covariance and $H_{k,i} = H$ in (7) at the $i^{th}$ iteration at the $k^{th}$ time step; $W_k = W$ in (2), $b_k = \bar{b}$ in (5) and $\bar{b}_k = \bar{b}$ in (6) at the $k^{th}$ time step:

$$P_{k,i} = (H_{k,i}^T W_k H_{k,i})^{-1}$$

$$\Delta \hat{x}_{k,i} = P_{k,i} H_{k,i}^T W_k (\bar{b}_k - \bar{b}_{k,i})$$

$$\hat{x}_{k,i+1} = \hat{x}_{k,i} + \Delta \hat{x}_{k,i}$$

(8)

Iterations stop when either 1) the states are no longer improved by the iteration, or 2) the number of iterations reaches the allowable limit. This GLSDC algorithm is robust when there are four or more beacons measured, except near certain geometric conditions that are avoided by careful beacon placement and by limiting the range of operation.

Optimal Nonzero Set Point Controller

The optimal Nonzero Setpoint (NZSP) is a command structure which controls the plant to a terminal steady-state condition, with guaranteed tracking properties. It is used to develop a
baseline autonomous controller according to Ref. 16 for evaluating the VisNav system in aerial refueling. For a linear time invariant system with $n$ states and $m$ controls,

$$
\begin{align*}
\dot{x} &= Ax + Bu ; \quad x(0) = x_0 \\
y &=Cx + Du \\
x &\in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^m
\end{align*}
$$

it is desired to command some of the initial outputs $y$ to steady-state terminal output values $y_m$ and keep them there as $t \to \infty$. If these terminal outputs are trim states, denoted by $\ast$, then at the terminal trim condition the system is characterized by

$$
\begin{align*}
\dot{x}^\ast &= Ax^\ast + Bu^\ast \equiv 0 \\
y_m &= Hx^\ast + Du^\ast \\
x^\ast &\in \mathbb{R}^n, u^\ast \in \mathbb{R}^m, y_m \in \mathbb{R}^m
\end{align*}
$$

For guaranteed tracking, the number of commanded outputs $y_m$ must be less than or equal to the number of controls $m$. Error states and error controls are defined as

$$
\begin{align*}
\tilde{x} &= x - x^\ast \\
\tilde{u} &= u - u^\ast
\end{align*}
$$

Where $\tilde{x}$ and $\tilde{u}$ are the error between the current state and control respectively, and the desired state and control respectively. The state equations can be written in terms of error states as

$$
\begin{align*}
\dot{\tilde{x}} &= \dot{x} - \dot{x}^\ast = Ax + Bu - (Ax^\ast + Bu^\ast) \\
\dot{\tilde{x}} &= A\tilde{x} + B\tilde{u}
\end{align*}
$$

with a cost function to be minimized
\[
J = \frac{1}{2} \left[ \dot{\mathbf{x}}^T Q \mathbf{x} + \mathbf{\tilde{u}}^T R \mathbf{\tilde{u}} \right] dt
\]

(11)

The optimal control \( \mathbf{\tilde{u}} \) which minimizes (10) is obtained by solving the matrix algebraic Riccati equation

\[
P A + A^T P - P B R^{-1} B^T P + Q = 0
\]

resulting in

\[
\mathbf{\tilde{u}} = -R^{-1} B^T P \dot{\mathbf{x}} = -K \dot{\mathbf{x}}
\]

A feedback control law in terms of the measured states is obtained by converting \( \mathbf{\tilde{u}} \) back to \( \mathbf{u} \), giving

\[
\mathbf{u} = \left( \mathbf{u}^* + Kx^* \right) - Kx
\]

(12)

with \( \mathbf{u}^* \) and \( x^* \) constants. They are solved for directly by inverting the quad partition matrix:

\[
\begin{bmatrix}
A & B \\
H & D
\end{bmatrix}^{-1} = \begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{x}^* \\
\mathbf{u}^*
\end{bmatrix} = \begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{y}_m \end{bmatrix}
\]

and solving for

\[
\mathbf{x}^* = X_{12} \mathbf{y}_m
\]

\[
\mathbf{u}^* = X_{22} \mathbf{y}_m
\]

Upon substitution in (11) control law implementation equation becomes

\[
\mathbf{u} = \left( X_{22} + KX_{12} \right) \mathbf{y}_m - Kx
\]

(13)

For the optimal control policy \( \mathbf{u} \) to be admissible, the quad partition matrix must be invertible. Therefore, the equations for \( x^* \) and \( u^* \) must be linearly independent, and the number of outputs
or states that can be driven to a constant value must be less than or equal to the number of available controls. An advantage of this controller is the guarantee of perfect tracking of a number of outputs equal to the number of controls, independent of the value of the gains, provided they are stabilizing. The gains can be designed using any desired technique, and only affect the transient performance, and not the guarantee of steady-state performance.

**Proportional Integral Filter with Control Rate Weighting**

The optimal NZSP controller developed above assumes that there are no exogenous inputs to the system. A controller for autonomous aerial refueling must possess both stability robustness and performance robustness, since it must operate in the presence of uncertainties, particularly unstructured uncertainties such as atmospheric turbulence. One technique to improve the disturbance rejection properties of a controller to exogenous inputs is to pre-filter the control commands with a low pass filter. This will also reduce the risk of Pilot Induced Oscillations (PIO) by reducing control rates. An effective technique which permits the performance of the pre-filter to be tuned with quadratic weights, called optimal Control Rate Weighting (CRW), is used for this purpose. It is developed here as part of the broader Proportional-Integral-Filter (PIF) methodology, as an extension of the optimal NZSP developed in (9) – (13). The resulting controller is termed Proportional Integral Filter - Nonzero Setpoint - Control Rate Weighting (PIF-NZSP-CRW).

For the present problem a type-1 system performance is desired, so integrator states $y_1$ are created such that body-axis velocities $u$ and $v$ are integrated to $x_{body}$ and $y_{body}$. To obtain the desired filtering of the controls, the rates of the controls are also added as states $u_1$. The optimal NZSP is extended into the optimal PIF-NZSP structure by first creating the integral of the commanded error
\[
\dot{y}_i = y - y_m; \quad \dot{y}_i \in \mathbb{R}^m
\]  

which upon substitution with (9) becomes

\[
\dot{y}_i = (Hx + Du) - y_m \\
= Hx + Du - Hx^* - Du^* \\
= H\ddot{x} + D\ddot{u}
\]

The augmented state-space system including the control rate states and integrated states is then

\[
\dot{\tilde{x}}_i = \begin{bmatrix} \tilde{x} \\ \tilde{u} \\ \tilde{y} \\ H \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ H & D & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{u} \\ \tilde{y} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} \begin{bmatrix} u \\ y \\ \ddot{u} \end{bmatrix}
\]

The quadratic cost function to be minimized has the form

\[
J = \frac{1}{2} \int_0^\infty \left[ \tilde{x}^T Q \tilde{x} + \tilde{u}^T R \tilde{u} + \tilde{u}^T S_{\text{rate}} \tilde{u} + \tilde{y}^T Q_2 \tilde{y} \right] dt
\]

where the matrix \( Q \) \in \mathbb{R}^{nxn} weights error states, the matrix \( R \) \in \mathbb{R}^{mxm} weights error controls, the matrix \( S_{\text{rate}} \) \in \mathbb{R}^{nxn} weights the control rates, and the matrix \( Q_2 \) \in \mathbb{R}^{nxp} weights the integrated states, with \( p \) the number of integrated states. Combining into the standard linear quadratic cost function form,

\[
J = \frac{1}{2} \int_0^\infty \begin{bmatrix} \tilde{x}_i^T & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & Q_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_i \\ u_i \end{bmatrix} dt
\]

The minimizing control \( \tilde{u}_i \) is obtained from the solution to the matrix algebraic Riccati equation
\[ PA + A^T P - PBR^{-1}B^T P + Q = 0 \]

resulting in

\[ \ddot{u}_f = -K_1\ddot{x} - K_2\dot{u} - K_3y_f \tag{15} \]

Re-writing (15) in terms of the measured state variables provides

\[ u_f = (u_f^* + K_1x^* + K_2u^*) - K_1x - K_2u - K_3y_f \tag{16} \]

with all * quantities constant, except for \( u_f^* \) which is equal to zero by the definition of steady-state. The constants \( x^* \) and \( u^* \) can be solved for by forming the quad partition matrix

\[
\begin{bmatrix} A & B \\ H & D \end{bmatrix}^{-1} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}
\]

\[
\begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \begin{bmatrix} 0 \\ y_m \end{bmatrix}
\]

and solving for

\[ x^* = X_{12}y_m \]
\[ u^* = X_{22}y_m \]

Upon substituting in (16) the control policy is

\[ u_f = (K_1X_{12} + K_2X_{22})y_m - K_1x - K_2u - K_3y_f \tag{17} \]

Note that this PIF-NZSP control policy requires measurement and feedback of the control positions, in addition to full state feedback, in order to be admissible. As with the NZSP, the gains can be determined using any desired technique provided they are stabilizing. In this paper, the gains are designed using linear quadratic methods, thereby providing optimal gains.
UAV Design and Simulation Model

The UAV model used for design and simulation purposes is called UCAV6. The UCAV6 simulation was developed in 1997 at Texas A&M University under an Office of Naval Research program into autonomous Uninhabited Combat Air Vehicles (UCAV). It is a roughly 60% scale AV-8B Harrier aircraft (Figure 1), with the pilot and support devices removed and the mass properties and aerodynamics adjusted accordingly. The simulation is a nonlinear, non real-time, six-degree-of-freedom computer code written in Microsoft Visual C++ 5.0. The UCAV6 longitudinal and lateral directional linear models used for both controller synthesis and simulation in this paper were obtained from the UCAV6 nonlinear simulation, and are described in detailed in Ref. 17.

Numerical Example

The example will exercise the PIF-NZSP-CRW controller connected to a high fidelity simulation of the VisNav sensor system. The GLSDC navigation solution provides the drogue position and attitude estimates directly to the PIF-NZSP-CRW controller. Process and sensor noise obtained from laboratory testing of the VisNav hardware is applied to the VisNav system simulation models. The controller uses full-state feedback, and all gains are designed using the optimal sampled-data technique. The PIF-NZSP-CRW controller is designed by first selecting state and control vectors of

\[
\begin{align*}
T_{ lon} & = \begin{bmatrix} u & w & q & \theta & \delta_e & \delta_{per} & x_{body} & z_{body} \end{bmatrix} \\
T_{ lat/a} & = \begin{bmatrix} v & p & r & \psi & \delta_a & \delta_r & y_{body} \end{bmatrix} \\
u & = \begin{bmatrix} \dot{\delta}_e & \dot{\delta}_{per} & \dot{\delta}_a & \dot{\delta}_r \end{bmatrix}
\end{align*}
\]

where \( p, q, r \) are body-axis angular velocities; \( \psi, \theta, \phi \) are the Euler attitude angles; \( \delta_a, \delta_e, \delta_r, \delta_{per} \) are the control effectors aileron, elevon, rudder, and throttle; and body-axis
positions \( x_{body}, y_{body}, z_{body} \) result from integration of the body-axis velocities \( u, v, w \). The basic NZSP structure permits a number of commanded outputs equal to the number of controls, so with four controls (elevon, aileron, rudder, throttle), the controller commands the UAV to the stationary drogue position \( (x_d, y_d, z_d) \), with a specified yaw attitude angle. The sample period is \( T = 0.1 \) seconds for all sub-systems in the combined system.

The control objective is to dock the tip position of the refueling probe with a stationary drogue receptacle with an accuracy of \( \pm 1.5 \) feet. The initial position of the drogue is 100 feet in front, 100 feet above, and 25 feet to the left of the initial trimmed position of the VisNav equipped UAV. An important requirement is to ensure that the probe engages the drogue with a forward velocity less than 10 feet per second. The drogue basket is configured with eight LED VisNav beacons. The four outer beacons are located 8 inches from the nozzle in a 1.18 foot diameter circle. This location corresponds to a typical location where LED lights are currently installed by drogue manufacturers for night time refueling operations. The inner four beacons are located at the nozzle itself in a circle with a diameter of 6 inches (Figure 7). The VisNav PSD sensor is mounted on the UAV, 35 feet behind the tip of the refueling probe and 5 inches above it. For this example, the VisNav relative position estimates are obtained from a simulation of the VisNav system that includes calibrations, range effects, corrections due to optical distortions, and sensor noise. The combined system of the PIF-NZSP controller with feedback measurements provided by simulations of the VisNav sensor is simulated for a flight condition of 250 KTAS at 20,000 feet altitude, in the presence of Dryden moderate turbulence corresponding to this flight condition (sigma gust=8).

Figures 8 though 12 show that the system performs well and meets all specifications. The desired low pass filtering effect of the PIF-NZSP is realized, as the controller commands the tip of the probe to the drogue smoothly, and with nearly zero steady-state error even in the presence
of moderate turbulence. The vertical and lateral errors are seen to converge smoothly and quickly, while the horizontal error converges only gradually as desired, so that the probe docks with the within the docking velocity specification (Figures 9 and 10). Angle-of-attack and sideslip angles excursions are small and well damped (Figure 11).

Figure 12 shows the throttle increased as the UAV climbs toward the drogue. Upon docking, the throttle is reduced back to the original trim setting as desired. The other control positions and rates are well within acceptable limits, and the control activity is relatively smooth despite the moderate turbulence.

Conclusions and Further Work

This paper presented the preliminary design of an accurate and reliable vision based sensor and controller for autonomous aerial refueling of unmanned air vehicles. The essential features of the navigation sensor and autonomous controller are discussed. The sensor makes use of closed structured light in a way that makes the navigation robust with respect to off-nominal lighting and system performance. The control system utilizes an optimal Proportional-Integral-Filter Nonzero Setpoint control law, which receives relative position measurements derived from a VisNav system of light emitting diode beacons, a position sensing diode sensor, and associated relative navigation algorithms. The system was simulated for the case of docking with a stationary drogue from an initial offset in three axes, in the presence of turbulence. Results indicate that the system is able to effectively accomplish the docking task within specifications of docking accuracy, docking velocity, and control effector position, rate, and activity. The disturbance rejection properties of the control system in turbulence are judged to be good and provide a basis for optimism as regards proceeding toward actual implementation. Further investigations are required to extend the controller structure to more effectively track time
varying commands, and to evaluate performance with the full suite of sensor dynamics and measurement error models applied. It is recommended that these studies be done in parallel with implementation for further laboratory and flight tests.

Acknowledgments

This research is funded by a National Defense Science and Engineering Graduate Fellowship in conjunction with the Army Research Office; by Sargent Fletcher Incorporated; by the State of Texas Advanced Technology Program; and by Zonta International Women’s Organization. This support is gratefully acknowledged by the authors.

Appendix: UCAV6 Linear Models

State-space non-parametric linear models (all angular quantities are in radians) as fully defined in Ref. 17. Flight Condition: 250 KTAS, 20,000 feet.

Longitudinal:
\[
\mathbf{x}_{\text{long}} = \begin{bmatrix} u & w & x_{\text{body}} & q & \theta & h \end{bmatrix}^T \quad \mathbf{u}_{\text{long}} = \begin{bmatrix} \delta_e & \delta_{\text{pwr}} \end{bmatrix}^T
\]

\[
A_{\text{long}} = \begin{bmatrix}
-0.0343 & 0.1618 & 0 & -31.9958 & -32.0198 & -0.0000 \\
-0.0658 & -1.3474 & 0 & 409.5377 & -2.4344 & -0.0003 \\
1.0000 & 0 & 0 & 0 & 0 & 0 \\
-0.0007 & -0.0225 & 0 & -0.7782 & 0 & -0.0000 \\
0 & 0 & 0 & 1.0000 & 0 & 0 \\
0.0758 & -0.9971 & 0 & 0 & 422.0000 & 0
\end{bmatrix}
\]

\[
B_{\text{long}} = \begin{bmatrix}
0.0081 & 0.2559 \\
0.2774 & 0.2286 \\
0 & 0 \\
0.1164 & 0.0143 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

Lateral/Directional:

\[
\mathbf{x}_{\text{lat/d}} = \begin{bmatrix} y_{\text{body}} & \nu & p & r & \phi & \psi \end{bmatrix}^T \quad \mathbf{u}_{\text{latD}} = \begin{bmatrix} \delta_u & \delta_r \end{bmatrix}^T
\]

\[
A_{\text{lat/d}} = \begin{bmatrix}
0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & -0.3326 & 31.9958 & -418.0678 & 32.0198 & 0 \\
0 & -0.0192 & -3.6427 & 1.7252 & 0 & 0 \\
0 & 0.0178 & -0.2158 & -1.1923 & 0 & 0 \\
0 & 0 & 1.0000 & 0.0760 & 0 & 0 \\
0 & 0 & 0 & 1.0029 & 0 & 0
\end{bmatrix}
\]

\[
B_{\text{lat/d}} = \begin{bmatrix}
0 & 0 \\
-0.2945 & 0.4481 \\
0.5171 & 0.0704 \\
0.0239 & -0.0895 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]
References


Figure 1. AV-8B Harrier Refueling Using the Probe-and-Drogue Method
Figure 2. Geometry and Axis Conventions for the Probe-and-Drogue Refueling Problem
Figure 3. Illustration of VisNav System and Geometries
Figure 4. VisNav System Architecture
Figure 5  VisNav sensor box (top), Position Sensing Diode sensor with pre-amplifiers (center), and LED beacon (bottom).
Figure 6. Conceptual VisNav LED Beacon Configuration on Drogue Basket
3 feet diameter

Figure 7. VisNav LED Beacon Configuration on Drogue Basket
Figure 8. Probe Tip Trajectory For Simulated Docking Maneuver
Figure 9. Probe Relative Position Errors and Attitudes
Figure 10. Perturbation Velocities and Accelerations
Figure 11. Body Axis Rates, Angle-of-Attack, and Sideslip Angle
Figure 12. Control Effector Positions and Rates