Structured Model Reference Adaptive Control For Vision Based Spacecraft Rendezvous And Docking

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A structured model reference adaptive control law has been developed for vision based spacecraft rendezvous and docking problem. A highly accurate relative motion of chaser spacecraft can be estimated by processing the reliable high bandwidth measurements of newly developed VISNAV sensor through a nonlinear geometric estimation process. The adaptive control law formulation is based upon the Lyapunov’s direct stability theorem and imposes the exact kinematic equations at velocity level while taking care of model uncertainties and disturbances at acceleration level. The essential ideas and results from computer simulations are presented to illustrate the algorithm developed in paper.

Introduction

Rendezvous and docking are essential parts for future autonomous space transportation missions such as ISS supply and repair. Autonomous proximity operations are required for a large number of other future mission concepts. For the docking of two spacecraft a highly precise position and attitude control is required which further requires precise measurements of the relative position and attitude of docking spacecrafts. These requirements lead to the identification, development and evaluation of several relative navigation sensors, such as VGS (Video Guidance Sensor), Global Positioning System (GPS) and others. Reliability, power, weight, accuracy, and cost of the sensors for the space missions determine the suitability of the sensor for a specific space mission.

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A new vision based sensor has been developed at Texas A&M university to address the well known difficulties which arise with video based vision navigation. The difficulties include: often inadequate spatial accuracy, slow frame rates, image processing computational burden, difficulties with lighting requirements, occasional failure of pattern recognition methods, etc. The new sensor has an analog detector in the focal plane with a rise time of a few microseconds. Accuracies of better than one part in 5000 of the field of view at a distance of 30m with an update rate of 100Hz are obtained by patented new electro-optical design. As a consequence, we can structure the light emitted by a set of actively controlled light emitting diodes (LEDs), in a fashion analogous to radar modulation/de-modulation, enabling easy rejection of ambient optical energy. The laser LEDs serve as actively commanded (cooperative vision) beacon targets, the line of sight toward these targets can be measured precisely. This enables, for example, six degree of freedom navigation with essentially zero image processing and no need for pattern recognition, as well as adaptively optimized signal to noise ratio for each measurement. Six degree of freedom navigation accuracies of < .1 inches in translation and < .01 degrees in orientation, updated at 100Hz are routinely possible by this approach.

Spacecraft rendezvous and docking require very precise translational and rotational maneuvers. These requirements necessitate the use of non-linear spacecraft dynamic models for control system design. We will consider a general asymmetric spacecraft containing 3 momentum wheels for attitude maneuvers, variable thrusters for translational maneuvers and nonlinear Clohessy-Wiltshire equations for the translation motion of the chaser spacecraft. The attitude motion of spacecraft will be represented by well known Euler’s equations for angular velocity evolution and attitude parameter kinematic equations. Although Clohessy-Wiltshire equations and Euler’s equations represent a near exact dynamical model, for control design purposes complications may arise from uncertain spacecraft inertia and mass properties which can change due to fuel consumption, solar array deployment, payload variation etc. Furthermore stability robustness due to model errors and disturbances are primary consideration for design of any autonomous control system.

To address all these problems, a structured model reference adaptive control law has been developed for the vision based spacecraft rendezvous and docking problem. The adaptive control law formulation normally requires full state measurements which requires, for
example additional sensors on-board like rate gyros to measure the angular velocities of the spacecraft. To avoid the need of additional sensors (beyond the vision sensor), the velocity level states can be “measured” (estimated) by effectively differentiating the position measurements.\textsuperscript{4} Crassidis et. al. has developed an optimal and efficient non-iterative algorithm for attitude and position determination from line of sight observations.\textsuperscript{5} This algorithm makes use of fast measurement rate(100Hz) of VISNAV sensors to calculate angular velocities and estimate linear velocities to propagate kinematic model. However, this kind of approach is based upon unproven separation principle for nonlinear systems and velocity estimates are known to be susceptible to high noise. In this paper, we introduce a purely geometric way of attitude determination from LOS observations which will allows us to decouple the controller design from the observer design. In lieu of designing a conventional Kalman filter like estimator to estimate velocity, we utilize a recently developed filter approach\textsuperscript{3} which effectively unifies observer and controller design methodology. In this paper, we will present the adaptive control law which incorporates the velocity generating filter from position measurements.\textsuperscript{6,7,8} The observer formulation and the adaptive control law formulation are based upon Lyapunov’s direct stability theorem and imposes the exact kinematic equations at velocity level while taking care of model uncertainties and disturbances at acceleration level.

The structure of this paper proceeds as follows. First a review of Clohessy-Wiltshire and Euler’s equation is shown followed by the review of measurement model for VISNAV sensor. Then, a review of geometric attitude and relative position estimation using VISNAV is given. Next, a partial state feedback adaptive controller is derived making use of geometric attitude estimates. Finally, the controller is tested using a simulated rendezvous and docking maneuver.

**Equations of Motion**

In this section, a review of nonlinear spacecraft dynamics is presented which includes the relative orbit dynamics, attitude kinematics and rotational dynamics using momentum transfer torque actuation with 3 momentum wheels.
Figure 1  LVLH and Body Frame

Coordinate System

The relevant coordinate systems are the Local-vertical-Local-Horizontal (LVLH) reference frame attached to the ‘target’ space vehicle and the orthogonal body frame fixed to the center of mass of the ‘chaser’ spacecraft, as shown in figure 1. The LVLH reference frame is attached to the center of mass of target space vehicle with $X$-axis pointing radially outward of its orbit, $Y$-direction perpendicular to $X$ along its direction of motion and $Z$ completes the right handed co-ordinate system. Usually in rendezvous and docking problem, LVLH attitude is taken as the reference target trajectory for the chaser spacecraft.
Relative Motion Dynamics

The motion of chaser w.r.t. the target spacecraft in the LVLH frame is described by the fully non-linear Clohessy-Wiltshire equations, given as follows:

\[ \ddot{x} - 2 \dot{\theta} \dot{y} - \dot{\theta}^2 x - 2 \frac{\mu}{r_c^3} x = - \frac{\mu (r_c + x)}{\rho^{3/2}} + \frac{\mu}{r_c^2} - 2 \frac{\mu}{r_c^3} x + \frac{F_x}{m} \]
\[ \ddot{y} + 2 \dot{\theta} \dot{x} + \dot{\theta}^2 y + \frac{\mu}{r_c^3} y = - \frac{\mu y}{\rho^{3/2}} + \frac{\mu}{r_c^3} y + \frac{F_y}{m}, \quad \rho = (r_c + x)^2 + y^2 + z^2 \]
\[ \ddot{z} + \frac{\mu}{r_c^3} z = - \frac{\mu z}{\rho^{3/2}} + \frac{\mu}{r_c^3} z + \frac{F_z}{m} \]
\[ \ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^3} \]
\[ \ddot{\theta} = -2 \frac{\dot{r}_c \dot{\theta}}{r_c} \]

where \( x, y, z \) represents the relative position of chaser w.r.t. target, \( r_c \) refers to the scalar radius of the target from the center of the Earth, \( \theta \) represents the latitude angle of the target, and \( \mu \) is the gravitational parameters. \( F_x, F_y \) and \( F_z \) are the control forces and \( m \) is the mass of chaser spacecraft. These amounts to the classical Encke differential equations of the chaser vehicle written in rotating LVLH coordinate system centered in the target vehicle.

Attitude Dynamics

The rotation motion of the chaser spacecraft in body frame is described by well known Euler’s equations of motion:

\[ \mathbf{I} \dot{\mathbf{\omega}} + [\bar{\omega}] \mathbf{I} \omega = \mathbf{\tau} \]  

where, \( \omega \in \mathbb{R}^3 \) represents the angular rates of chaser in the chaser vehicle body frame, \( \mathbf{I} \in \mathbb{R}^{3 \times 3} \) is the symmetric positive definite inertia matrix and \( \mathbf{\tau} \in \mathbb{R}^3 \) represents the external torques acting on the spacecraft. \([\bar{\omega}]\) is the skew-symmetric matrix that represents the cross product of two vectors and can be written as:

\[ [\bar{\omega}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \]
The effective external control torque acting on the spacecraft is generally generated by one or more momentum wheels. Euler’s equations of motion can be augmented with wheel dynamics for a general asymmetric spacecraft containing 3 momentum wheel\textsuperscript{10} and written as:

\[ I \dot{\omega} + [\tilde{\omega}] I \omega + A \dot{\Omega} + [\tilde{\omega}] A \Omega = 0 \]  

where, \( A \in \mathbb{R}^{3\times3} \) represents the wheel moment of inertia, \( \Omega \in \mathbb{R}^3 \) represents the wheel angular rate. The fixed two gyroscope coupling terms represent the effective torque on the chaser vehicle.

The attitude of chaser w.r.t. to target is described by using a 3 \times 1 vector \( \sigma \) of modified Rodriguez parameters (MRP).\textsuperscript{9} The exact kinematic differential equations in terms of the MRP vector \( \sigma \) can be written as:

\[ \dot{\sigma} = \frac{1}{4} J(\sigma) \omega \]  

where the kinematic operator \( J(\sigma) \) can be written as:

\[ J = (1 - \sigma^T \sigma) I_{3\times3} + 2 [\tilde{\sigma}] + 2 \sigma \sigma^T \]  

We mention that the rows (columns) of \( J \) are orthogonal vectors; a unique property amongst all known minimal coordinate description of orientation.

**Measurement Model and Attitude Determination**

In this section, a short review of collinearity equations for attitude and position determination for the VISNAV sensor is presented.\textsuperscript{1} Finally, a geometric attitude determination algorithm is also presented.
The attitude and position from LOS observations are determined by using following collinearity equations:

\[
x_i = -f \frac{C_{11}(X_i - X_c) + C_{12}(Y_i - Y_c) + C_{13}(Z_i - Z_c)}{C_{31}(X_i - X_c) + C_{32}(Y_i - Y_c) + C_{33}(Z_i - Z_c)} + x_0, \quad i = 1, 2, \ldots, N \tag{7}
\]

\[
y_i = -f \frac{C_{21}(X_i - X_c) + C_{22}(Y_i - Y_c) + C_{23}(Z_i - Z_c)}{C_{31}(X_i - X_c) + C_{32}(Y_i - Y_c) + C_{33}(Z_i - Z_c)} + y_0, \quad i = 1, 2, \ldots, N \tag{8}
\]

where, \(C_{ij}\) are the unknown elements of attitude matrix \(C\) associated to the orientation of the image plane with respect to the reference plane, \(f\) is known focal length, \((x_i, y_i)\) are the known image space measurements for the \(i^{th}\) LOS, \((X_i, Y_i, Z_i)\) are the known object space locations (target spacecraft) of the \(i^{th}\) LOS, \((X_c, Y_c, Z_c)\) are the unknown object space location (chaser spacecraft) of the sensor and \(N\) is the total number of measurements. \(x_0\) and \(y_0\) refers to the principal point offset. We mention that a rigorous calibration process compensates for all non ideal effects, so that after calibration corrections, the collinearity equations of equations (7) and (8) are a valid model. Figure 2 shows the object space and image space coordinate frames for the PSD sensors and beacons.
The sensor observations can also be written in following orthogonal projection:

\[ \mathbf{b}_i = \mathbf{Cr}_i, \ i = 1, 2 \cdots , N \quad (9) \]

where, \( \mathbf{b}_i \) and \( \mathbf{r}_i \) can be written as:

\[
\mathbf{b}_i = \begin{bmatrix}
    x_0 - x_i \\
    y_0 - y_i \\
    f
\end{bmatrix}
\quad (10)
\]

\[
\mathbf{r}_i = \begin{bmatrix}
    X_i - X_c \\
    Y_i - Y_c \\
    Z_i - Z_c
\end{bmatrix}
\quad (11)
\]

Given measurements \((x_i, y_i)\) for four or more beacons located at known target vehicle points \((X_i, Y_i, Z_i)\), equations (7) and (8) or equivalently, equations (9)-(11) can be inverted by least square differential correction.\(^{11}\) Since these are algebraic equations and the measurements are effectively instantaneous, this least square process effectively establishes a purely geometric estimate of relative position and orientation of the chaser vehicle with respect to a coordinate system fixed in the target vehicle.

**Adaptive Control Law Formulation**

In this section, the velocity free adaptive control law will be derived for translation and rotational motion maneuver, using a Lyapunov’s direct stability theorem. This control law is inspired by the work of Tsiotras,\(^{12}\) and Akella.\(^{7}\)

**Adaptive Control Formulation For Relative Translation Motion**

In this section, we seek to design an adaptive feedback control law for the system described by equation(1) to track a given reference trajectory specified by \(\mathbf{x}_r\). It should be noticed that equation(1) can be rewritten in following form:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= A_1x_1 + A_2x_2 + g(x_1) + \frac{F}{m}
\end{align*}
\quad (12)
\]

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where, $\mathbf{x}_1 = [x \ y \ z]^T$ and $\mathbf{x}_2 = [\dot{x} \ \dot{y} \ \dot{z}]^T$ refer to relative position and velocity variables. The matrices $A_1$, $A_2$ and vector $g$ can be constructed as follows:

$$A_1 = \begin{bmatrix} \ddot{\theta}^2 + \frac{2 \mu}{r_c^2} & \dot{\theta} & 0 \\ -\dot{\theta} & \dot{\theta}^2 - \frac{\mu}{r_c^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(13)

$$A_2 = \begin{bmatrix} 0 & 2 \dot{\theta} & 0 \\ -2 \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(14)

$$g = \begin{bmatrix} -\frac{\mu (r_c + x)}{r_c^2} + \frac{\mu}{r_c^2} - 2 \frac{\mu}{r_c^2} x \\ -\frac{\mu y}{r_c^2} + \frac{\mu y}{r_c^2} \\ -\frac{\mu z}{r_c^2} + \frac{\mu z}{r_c^2} \end{bmatrix}$$

(15)

If we denote the relative position and velocity tracking error by $e_1$ and $e_2$ respectively, then it is easy to show that the error dynamics can be written as:

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = A_1 x_1 + A_2 x_2 + g(x_1) + \frac{F}{m} - \dot{x}_r$$

(16)

Now, we will augment the system described by equation (16) with a “lead filter” similar to one that has been proposed in references\(^2\)\(^3\). The filter generates the pseudo velocity estimates and is governed by following differential equation:

$$\dot{z} = A_m z + e_1$$

(17)

where, $z \in \mathbb{R}^3$ and $A_m$ is any prescribed Hurwitz matrix which satisfies the following Lyapunov equation.

$$A_m^T P + PA_m = -Q$$

(18)

where, both $P$ and $Q$ are symmetric positive definite matrices.

**Theorem 1.** For the system described by equations (16) and (17) with no information on
spacecraft mass $m$, if the following control input $F$ is adopted,

$$F = -\dot{m}(P e_1 + P(A_m z + e_1) + A_1 x_1 + A_2 \dot{x}_r + g(x_1) - \dot{x}_r) \quad (19)$$

with the estimates $\dot{m}$ being updated by following adaptive law

$$\dot{m} = -\Gamma_1 e_2^T \frac{F}{m} \quad (20)$$

where $\Gamma_1 > 0$ is a gain that controls the rate of mass parameter learning. Then, we can ensure that $e_1 \to 0$ and $e_2 \to 0$ as $t \to \infty$.

Proof. Let $\dot{m} = m - \dot{m}$ and let us consider the following Lyapunov function:

$$V = \frac{1}{2} e_1^T P e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} \dot{z}^T P \dot{z} + \frac{1}{2} \dot{m} \Gamma_1^{-1} \dot{m} \quad (21)$$

It should be noticed that $V$ is a radially unbounded positive definite function. The time derivative of $V$ along the system trajectories is given by

$$\dot{V} = e_1^T P \dot{e}_1 + e_2^T \dot{e}_2 + \frac{1}{2} \dot{z}^T P \dot{z} + \frac{1}{2} \ddot{z}^T P \dot{z} + \dot{m} \Gamma_1^{-1} \dot{m} \quad (22)$$

Now, using equations (16), (17), (19) and (20), we can rewrite equation (22) as follows:

$$\dot{V} = -\frac{1}{2} \dot{z}^T Q \dot{z} - e_2^T m^{-1} \dot{m} F + \dot{m} \Gamma_1^{-1} \dot{m} \quad (23)$$

Now using the fact that $\dot{m} = m - \dot{m}$, we can show that choosing the adaptive law of equation (20) yields

$$\dot{V} = -\frac{1}{2} \dot{z}^T Q \dot{z} \leq 0 \quad (24)$$

Since $\dot{V} \leq 0$ and $V > 0$, $\dot{V}$ is only negative semi-definite. However, we can easily show that $e_1, e_2, \dot{z} \in L_\infty$. Further from the integral of equation (24), it follows that $\dot{z} \in L_\infty \cap L_2$ and therefore from Barbalat’s Lemma $\dot{z} \to 0$ as $t \to \infty$, which in turn leads to $e_2 \to 0$ and to $\dot{m} \to 0$ based on equation (20). Similarly, considering higher derivatives we can show that $\dot{e}_2 \to 0$. Therefore, by using LaSalle’s invariance principle we can show that $e_1 \to 0$ and
\( e_2 \rightarrow 0 \) as \( t \rightarrow \infty \)\(^{13,8} \). Further from equation (17), we can conclude that \( z \rightarrow 0 \) as \( t \rightarrow \infty \). \(\square\)

It should be noted that the update law defined in equation (20) depends upon the unknown vector \( e_2 \) therefore for actual implementation of control law given in equation (19), the following equivalent equation should be used, which can be integrated to get the estimated mass parameter.

\[
\dot{\hat{m}}(t) = \hat{m}(0) + \Gamma_1 \int_0^t \frac{F}{\hat{m}} \hat{e}_1 dt
\]  

(25)

Adaptive Control Formulation For Rotation motion Maneuver

In this section, we present the adaptive control law formulation for attitude control. By utilizing thrusters for translational control and reaction wheels for attitude control, we can uncouple translational and rotational control to a high degree of approximation. We seek a control law for system described by equations (4) and (5) to track a given trajectory specified by \( \sigma_r \). This control law formulation is similar to one presented by Maruthi\(^7\) and Tanygin.\(^8\) The main new feature of the control law presented in this paper is that it also addresses the uncertainties in the modeling of the reaction wheel torque actuators (momentum wheels) dynamics due to unknown inertia matrix or misalignment with spacecraft axes.

Let us introduce three reference frame \( \mathcal{N} \), \( \mathcal{B} \) and \( \mathcal{R} \) to address the problem of attitude control. The reference frame \( \mathcal{N} \) corresponds to inertial frame fixed to the center of earth while reference frames \( \mathcal{B} \) and \( \mathcal{R} \) denotes the Body fixed and LVLH reference axes. Further, the MRP parameters corresponding to the orientation of LVLH frame with respect to the inertial frame are denoted by \( \sigma_r \) while those corresponding to the orientation of body frame with respect to inertial frame are denoted by \( \sigma \). Then the unit vectors in these three frames can be related by following relationships:

\[
\hat{b} = C(\sigma)\hat{n}
\]  

(26)

\[
\hat{r} = C(\sigma_r)\hat{n}
\]  

(27)

\[
\hat{b} = C(e)\hat{r}, \ C(e) = C(\sigma)C^T(\sigma_r)
\]  

(28)

where, \( e \) refers to the error MRP, that parameterize the attitude error between body frame
and the LVLH frame. Notice \( e \neq \sigma - \sigma_r \), unless the equations are linearized. The attitude matrix \( C(\sigma) \) in terms of MRP can be written as

\[
C(\sigma) = I - 4 \frac{1 - \sigma^T \sigma}{(1 + \sigma^T \sigma)^2} [\tilde{\sigma}] + \frac{8}{(1 + \sigma^T \sigma)^2} [\tilde{\sigma}]^2 \tag{29}
\]

The angular velocity error in body frame can be represented as:

\[
\delta \omega = \omega - \eta, \quad \eta = C(e) \omega_r \tag{30}
\]

Now, using the transport theorem,\(^9\) we can rewrite the governing differential equations (4) and (5) in terms of error angular rate and error MRP as follows:

\[
\dot{e} = \frac{1}{4} J(e) \delta \omega \tag{31}
\]

\[
I \delta \omega = - [\tilde{\omega}] I \omega - A \dot{\Omega} - [\tilde{\omega}] A \Omega + d - I [C(e) \omega_r - [\tilde{\omega}] \eta] \tag{32}
\]

Now, we again augment the error dynamics with a lead filter analogous to the one presented in the previous section.

\[
\dot{z} = A_m z + e \tag{33}
\]

\[
A_m^T P + PA_m = -Q \tag{34}
\]

Again \( A_m \) is a chosen Hurwitz matrix whereas \( P \) and \( Q \) are positive definite symmetric matrices.

**Theorem 2.** For the system described by equations (31), (32) and (17) with no information is used on the assumed unknown spacecraft mass inertia and wheel inertia matrices \( I \) and \( A \) respectively, if the following control input \( \dot{\Omega} \) is adopted,

\[
\dot{\Omega} = \hat{A}^{-1} \left[ \hat{d} + \frac{1}{4} J^T(e) P \dot{z} + \frac{1}{4} J^T(e) P e - \hat{I} C(e) \omega_r - [\hat{\eta}] \left( \hat{A} \Omega + \hat{I} \eta \right) \right] \tag{35}
\]
with the estimates \( \hat{I}(t) \) and \( \hat{A}(t) \) are being updated by following adaption law

\[
\dot{\theta} = \Gamma_1 W_d^T \delta \omega
\]  

(36)

where \( \theta \in \mathbb{R}^{15} \), is a parameter vector consisting of uncertain spacecraft inertia, wheel inertia and disturbance terms:

\[
\theta = \left\{ I_{11} \ I_{12} \ I_{13} \ I_{22} \ I_{23} \ I_{33} \ A_{11} \ A_{12} \ A_{13} \ A_{22} \ A_{23} \ A_{33} \ d_1 \ d_2 \ d_3 \right\}^T
\]  

(37)

\( W_d \in \mathbb{R}^{3 \times 15} \) is a regressor matrix.\(^7\)

\[
W_{d1} = \begin{bmatrix}
  cw_1 & cw_2 & cw_3 & 0 & 0 & 0 \\
  0 & cw_1 & 0 & cw_2 & cw_3 & 0 \\
  0 & 0 & cw_1 & 0 & cw_2 & cw_3
\end{bmatrix}, \quad cw = C(e)\dot{\omega}_r
\]  

(38)

\[
W_{d2} = \begin{bmatrix}
  0 & -\Omega_1 \eta_3 & \Omega_1 \eta_2 & -\Omega_2 \eta_3 & -\Omega_2 \eta_2 + \Omega_1 \eta_2 & \Omega_3 \eta_2 \\
  \Omega_1 \eta_3 & \Omega_2 \eta_3 & \Omega_3 \eta_3 - \Omega_1 \eta_1 & 0 & -\Omega_2 \eta_1 & -\Omega_3 \eta_1 \\
  \Omega_1 \eta_2 & -\Omega_2 \eta_2 + \Omega_1 \eta_1 & -\Omega_3 \eta_2 & \Omega_2 \eta_1 & \Omega_3 \eta_1 & 0
\end{bmatrix}
\]  

(39)

\[
W_{d3} = \begin{bmatrix}
  0 & -\eta_1 \eta_3 & \eta_1 \eta_2 & -\eta_2 \eta_3 & -\eta_3 \eta_3 + \eta_2 \eta_2 & \eta_3 \eta_2 \\
  \eta_1 \eta_3 & \eta_2 \eta_3 & \eta_3 \eta_3 - \eta_1 \eta_1 & 0 & -\eta_2 \eta_1 & -\eta_3 \eta_1 \\
  \eta_1 \eta_2 & -\eta_2 \eta_2 + \eta_1 \eta_1 & -\eta_3 \eta_2 & \eta_2 \eta_1 & \eta_3 \eta_1 & 0
\end{bmatrix}
\]  

(40)

\[
W_{d4} = \begin{bmatrix}
  u_1 & u_2 & u_3 & 0 & 0 & 0 \\
  0 & u_1 & 0 & u_2 & u_3 & 0 \\
  0 & 0 & u_1 & 0 & u_2 & u_3
\end{bmatrix}, \quad u = \dot{\Omega}
\]  

(41)

\[
W_d = [-W_{d1} - W_{d3} - W_{d2} - W_{d4} \ I_{3 \times 3}]
\]  

(42)

and \( \Gamma_1 \in \mathbb{R}^{15 \times 15} \) is any positive definite symmetric matrix. Then, we can ensure that \( e \to 0 \) and \( \delta \omega \to 0 \) as \( t \to \infty \).

**Proof.** Let us consider following Lyapunov function

\[
V = \frac{1}{2} e^T Pe + \frac{1}{2} \delta \omega^T P \delta \omega + \frac{1}{2} \dot{\theta}^T \Gamma_1^{-1} \dot{\theta}
\]  

(43)
Again $V$ is radially unbounded positive definite function. The time derivative of $V$ along the system trajectories is given by

$$\dot{V} = e^T P \dot{e} + \delta \omega^T I \delta \dot{\omega} + \frac{1}{2} \dot{z}^T P \dot{z} + \frac{1}{2} \ddot{z}^T P \ddot{z} + \bar{\theta}^T \Gamma_1^{-1} \bar{\dot{\theta}}$$  \hspace{1cm} (44)$$

Now, using equations (31), (32), (35),(33), and (34), we can rewrite equation (44) as follows:

$$\dot{V} = \delta \omega^T [-\bar{\eta}] \bar{\Theta} + [\bar{\eta}] \bar{I} \eta + \bar{d} - \bar{I} C(e) \dot{\omega} - \bar{A} u + \delta \omega^T [-\bar{\omega}] I \omega + [\bar{\omega}] \delta \omega \hspace{1cm} (45)$$

$$+ \ I[\bar{\omega}] \eta - \frac{1}{2} \dot{z} Q \dot{z} + \bar{\theta}^T \Gamma_1^{-1} \bar{\dot{\theta}} \hspace{1cm} (46)$$

Adopting the control law for $\dot{\bar{\theta}}$ from equation (36) in equation (45), we get

$$\dot{V} = - \frac{1}{2} \dot{z} Q \dot{z} + \delta \omega^T [-\bar{\omega}] I \omega + [\bar{\omega}] \eta + [\bar{\eta}] \bar{I} \eta \hspace{1cm} (47)$$

Using equation (30), it can be proved that the second term on right hand side is identically zero. Therefore, the equation for $\dot{V}$ reduces to following equation

$$\dot{V} = - \frac{1}{2} \dot{z}^T Q \dot{z} \leq 0 \hspace{1cm} (48)$$

Since $\dot{V} \leq 0$ and $V > 0$, we can easily show that $e$, $\delta \omega$, $\dot{z}$, $\bar{\theta} \in L_\infty$. Further from the integral of equation (48), it follows that $\dot{z} \in L_\infty \cap L_2$ and therefore from Barbalat’s Lemma $\dot{z} \to 0$ as $t \to \infty$, which in turn leads to $\dot{e} \to 0$ and to $\delta \omega \to 0$ based on equations (31). Similarly, considering higher derivatives we can show that $\delta \dot{\omega} \to 0$ and then by using LaSalle’s invariance principle we can show that $e \to 0$ and $\delta \omega \to 0$ as $t \to \infty$. Therefore from equation (33), we can also conclude that $z \to 0$ as $t \to \infty$.

Once again, it should be noted that the update law defined in equation (36) depends upon the unknown vector $\delta \omega$ therefore for actual implementation of control law given in equation (35), the following equivalent equation should be used, which can be integrated to get the estimated inertia parameters.

$$\bar{\theta}(t) = \bar{\theta}(t)(0) + \Gamma_1 \int_0^t W_d^T J^{-1}(e) \dot{e} dt \hspace{1cm} (49)$$
Further, it should be noticed that the control laws presented in this paper do not guarantee the convergence of unknown mass and inertia parameters to their true value but ensure that the parameter estimation errors are bounded. The convergence of unknown parameters to their true value can only be guaranteed by satisfying the persistence of excitation condition. Satisfactory qualitative convergence may be achieved without persistence of excitation in some examples.

**Numerical Simulations**

The control laws presented in this paper are illustrated in this section for a particular rendezvous and docking maneuver. It is assumed that chaser is at a distance of \(\{−100, −70, −80\}^T m\) from target spacecraft. The target spacecraft is assumed to be in circular orbit at an altitude of 400 km. The actual mass of chaser is assumed to be 50 kg with following inertia matrix:

\[
I = \begin{bmatrix}
250 & 100 & 50 \\
100 & 200 & 30 \\
50 & 30 & 150 \\
\end{bmatrix}
\]

The chaser is assumed to be equipped with 3 momentum wheel gyros for attitude control with following true wheel control influence matrix:

\[
A = \begin{bmatrix}
10 & 6 & 5 \\
6 & 12 & 4 \\
5 & 4 & 15 \\
\end{bmatrix}
\]

The reference target trajectory for translation motion of chaser is generated in a smooth ad hoc fashion by connecting a 3\(^{rd}\) order spline curve between initial state to the final desired position (\(\{0, 0, 0\}^T m\)) of chaser. To fit the smooth curve, the docking time is assumed to be 10 min. For rotation motion, the reference (target) trajectory is also generated by fitting a 3\(^{rd}\) order spline curve between desired initial orientation and desired final orientation of \(\{-\frac{\pi}{6}, -\frac{\pi}{9}, -\frac{\pi}{18}\}\) and \(\{\frac{\pi}{6}, \frac{\pi}{9}, \frac{\pi}{18}\}\) respectively.

Initially an error of 1 m is introduced for translation motion initial condition and initial attitude is set off from desired one by 50 %. All the initial conditions and tuning parameters
of controllers are set as given below:

**Translation Motion Initial Conditions:**

\[
x_1(0) = [-100, -70, -80]^T m, \ z(0) = [0 0 0]^T
\]

\[
m(0) = 40 kg, \ an \ error \ of \ 20\%
\]

\[
A_m = -I_{3\times3}, \ Q = 0.1 \times I_{3\times3}, \ \Gamma_1 = 5
\]

**Rotation Motion Initial Conditions:**

\[
I(0) = O_{3\times3}, \ z(0) = [0 0 0]^T
\]

\[
A_m = -0.09I_{3\times3}, \ Q = 10I_{3\times3}
\]

\[
A(0) = \begin{bmatrix}
1 & 0.6 & 0.5 \\
0.6 & 1.2 & 0.4 \\
0.5 & 0.4 & 1.5
\end{bmatrix}
\]

\[
\Gamma_1 = \begin{bmatrix}
4 \times 10^4 I_{6\times6} & O_{4\times3} \\
O_{3\times3} & 0.05I_{3\times3}
\end{bmatrix}
\]

The tuning parameters for control law were selected by trial and error to illustrate the essential ideas. But it is worthwhile to mention that the robustness of control law depends upon the choice of \(A_m\) and \(Q\) matrix. If we choose large value for \(Q\) and small value for \(A_m\) then we will get very robust control with respect to uncertainties in system dynamics but at the expense of large control effort and smoothness of control and state behavior.

For simulation purposes, the position and attitude estimates are assumed to be available at frequency of 100 Hz.

Figures (3) and (4) show the translation motion position tracking error (\(e_1\)) and velocity tracking error (\(e_2\)). From these figures, we can conclude that adaptive control law without velocity measurements presented in this paper performs well even in presence of high uncertainty in the mass of chaser spacecraft. The estimated mass of chaser and corresponding control forces are shown in figures (5) and (6) respectively. From figure (5), it should be noticed that the estimated mass of chaser converges to its true value.
Figures (7) and (8) show the attitude tracking error ($e$) and angular velocity tracking error ($\delta\omega$) for the simulated maneuver. The corresponding commanded wheel velocity is shown in figure (9) and the estimated spacecraft inertia parameters are shown in figure (10). From these figures, it is clear that adaptive control law presented in this paper is able to take care of high uncertainties in spacecraft inertia matrix and momentum wheel inertia matrix.

To illustrate the real capability of adaptive control law in presence of uncertainties and disturbances we design an highly demanding attitude maneuver. The reference attitude trajectory is assumed to be $\sigma_r = [0.5 \cos 0.2t \ 0.5 \sin 0.2t \ \sqrt{3} \frac{t}{T}]^T$. beside this we also simulated the effect of disturbance torque acting on chaser of frequency between 2 Hz and 30 Hz. The initial conditions are identical to those selected before. Figures (11) and (12) show the attitude tracking error ($e$) and angular velocity tracking error ($\delta\omega$) for this attitude maneuver. The corresponding commanded wheel velocity is shown in figure (13) and the estimated spacecraft inertia parameters are shown in figure (14). It must be noted that tracking errors in this case are an order of magnitude better when adaptation is on. From these results we can conclude that control law has the capability to handle the disturbance torques due to thruster misalignment, environment etc. The control effort required in this case can be smoothen by more judicious choice of tuning parameters of controller.

Concluding Remarks

An asymptotically stable partial state feedback adaptive controller has been designed for spacecraft rendezvous and docking maneuver. A geometric attitude estimation algorithm has also been presented for precise estimation of relative motion of chaser spacecraft from high bandwidth measurements of newly developed VISNAV sensor at Texas A&M university. The key point of the control presented in this paper is the consideration of actuator dynamics uncertainty by assuming momentum wheel inertias to be unknown. The control law presented in this paper is shown to work well even in presence of bounded disturbances and large parameter errors. However, the simulation results presented in this paper merely illustrate the capability of control for a particular rendezvous and docking maneuver, so much more testing would be required to reach any conclusions about the utility of this control law.
References


4Singla, P., “A New Attitude Determination Approach using Split Field of View Star Camera,” *Masters Thesis report*, Aerospace Engineering, Texas A&M University, College Station, TX, USA.


Figure 3  Position Tracking Error vs Time.

Figure 4  Velocity Tracking Error vs Time.
Figure 5  Estimated Mass vs Time.

Figure 6  Required Control Effort vs Time.
Figure 7  Attitude Tracking Error vs Time.

Figure 8  Angular Velocity Tracking Error vs Time.
Figure 9  Commanded Wheel Velocity (rad/sec) vs Time.

Figure 10  Estimated Spacecraft Inertia Parameters vs Time.
Figure 11  Attitude Tracking Error vs Time.

Figure 12  Angular Velocity Tracking Error vs Time.
Figure 13  Commanded Wheel Velocity (rad/sec) vs Time.

Figure 14  Estimated Spacecraft Inertia Parameters vs Time.