MORPHING AIRFOIL SHAPE CHANGE OPTIMIZATION WITH MINIMUM ACTUATOR ENERGY AS AN OBJECTIVE

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Abstract
Morphing aircraft are multi-role aircraft that use innovative actuators, effectors, and mechanisms to change their state to perform select missions with substantially improved system performance. State change in this study means a change in the cross-sectional shape of the wing itself, not chord extension or span extension. Integrating actuators and mechanisms into an effective, light weight structural topology that generates lift and sustains the air loads generated by the wing is central to the success of morphing, shape changing wings and airfoils. The objective of this study is to explore a process to link analytical models and optimization tools with design methods to create energy efficient, lightweight wing/structure/actuator combinations for morphing aircraft wings. In this case, the energy required to change from one wing or airfoil shape to another is used as the performance index for optimization while the aerodynamic performance such as lift or drag is constrained. Three different, but related, topics are considered: energy required to operate articulated trailing edge flaps and slats attached to flexible 2D airfoils; optimal, minimum energy, articulated control deflections on wings to generate lift; and, deformable airfoils with cross-sectional shape changes requiring strain energy changes to move from one lift coefficient to another. Results indicate that a formal optimization scheme using minimum actuator energy as an objective and internal structural topology features as design variables can identify the best actuators and their most effective locations so that minimal energy is required to operate a morphing wing.

Background
A morphing aircraft is a multi-role aircraft that, through the use of morphing technologies such as innovative actuators, effectors or mechanisms, changes its state substantially to complete all roles - with superior system performance.¹ For instance, for a hunter-killer aircraft, role A is long-duration loiter while role B is high-speed dash and role C is some form of energy deposition to neutralize the target. Morphing aircraft are a major part of a system that requires technology integration to manipulate geometric, mechanical electromagnetic or other mission critical features - on the ground or in-flight - to match vehicle performance to a well-defined environment and mission objective.

Modern aircraft already contain complex morphing devices to allow them to balance one mission demand against another but still perform a mission well. The simplest example is the use of landing and take-off flaps to allow transport aircraft to operate from shorter length airports and still cruise at high speed. The trade-off that favors morphing is the fuel saved when a smaller wing is used for efficient high-speed flight and deployable flaps generate increased lift at low landing and take-off speeds. The cost of this system performance balancing is expressed in metrics such as weight, complexity and cost.

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Morphing Aircraft Potential
combining many systems into one

Morphing shape/state changes are also necessary when long loiter time is coupled with a high-speed dash requirement or when stealth features are not required during the entire mission. In the latter case, morphing into a “faceted brick” shape like the F-117 for the stealthy portion of the mission would then change the aircraft radar cross-section “state” while preserving aerodynamic performance during long cruise segments.

Modern morphing aircraft concepts address system problems such as that depicted in Figure 1. The upper part of Figure 1 shows the different sensor aircraft required to identify targets for combat aircraft to engage and attack. In addition to the variety of aircraft used, this “observe, identify, target and attack” system requires command and control as well as human operators. The result is a complex system with large time lags that impede its effectiveness. The lower part of Figure 1 shows nature’s answer to a sensing and attack system. All capability, including system operation, is concentrated onboard the vehicle.

Wing morphing cannot be applied to all systems and all missions. In fact, it requires a major effort to identify the new missions that might result from modern morphing. However, once these missions and system architectures are identified and the aircraft that use them are developed, the result will be to create “game-changers.”

Minimizing actuator energy required to generate lift
Modern morphing concepts include wings with a variety of moving surfaces, such as: articulated flaps and slats; surface flow control devices; and continuously deforming surfaces. In the latter case, the deformation of the surface is generated either by internal elements that exert forces and moments on the aero structure to deform it or external devices such as continuously deforming trailing edge surfaces. The latter case includes Active Aeroelastic Wing concepts (AAW).²

Because wings are lightweight, structural flexibility and aeroelasticity are essential features of morphing wing design. The energy required to change wing cross-sectional shape to generate lift has two parts, the energy required to strain the structure and the energy required to move against the air pressures on the wing surface. The strain energy stored during the lift generation is wasted energy since it cannot be recovered by the power system onboard the aircraft. In fact, in some cases, such as aileron induced distortion that leads to aileron reversal, all of the energy goes to strain the structure and change the pressure distribution, but the result is no net lift and thus there is no useful output despite a large energy expenditure.

We will consider three related aero-structure interaction studies to illustrate how actuation energy plays a role in some morphing wing design activities. The first of these studies is determining the energy required to operate leading and trailing edge devices with different flap-to-chord ratios at different airspeeds. This study provides a useful illustration in how external aerodynamic forces can be used to assist the onboard control energy to generate lift. The second study illustrates the interaction between aeroelastic effects and energy required to generate lift on a flexible wing. This study has two parts, one with a simple strip theory model and the other with a more accurate aerodynamic configuration. The third study looks at a continuously deforming surface and focuses on the strain energy required to change from one airfoil shape to another, but without including aeroelastic effects. These airfoil shapes are limited to NACA 4-series airfoils to ensure that all of the shapes considered are smooth. A second part of the third study examines a set of shell-like finite element models and illustrates how the combination of actuator placement,
deployment and structural design can make a large difference in the energy required to operate morphing systems.

**Minimizing flap deflection energy for an idealized 2-DOF System**

The first of our three studies determines the energy required to operate leading and trailing edge devices with different flap-to-chord ratios at different airspeeds to generate lift. A 2-DOF model with pitch (torsion) freedom, $\theta$, about a hypothetical elastic axis, and a control deflection, $\delta_o$, is illustrated in Figure 2.

The airfoil chordwise pressure distribution at three different dynamic pressures is shown in Figure 2 for the same aileron input. The integral of the pressure distribution over the entire surface – the lift – decreases as airspeed increases and becomes negative when the aileron reverses. This is due to aeroelasticity. The hinge moment created by the aerodynamic forces on the aileron aft of the hinge line also changes as dynamic pressure increases. Figure 2 indicates that the moment about the hinge line, due to integrated pressure distribution about the hinge line, declines as airspeed increases.

The pressure distribution (positive up) on the airfoil is the linear superposition of the pressure distribution (positive upward) due to the aileron deflection (positive down, as in the figure) written as $p_{\text{due to } \delta} = p_\delta \delta$ and a twist angle $\theta$ (positive nose-up) that occurs when the aileron is deflected. This is written as $p_{\text{due to } \theta} = p_\theta \theta$. Static equilibrium considerations lead to the relation between $\theta$ and $\delta_o$ given as $\theta = f \delta_o$ where

$$f = \left(\frac{-q}{1-q}\right) \left(1 - \frac{q_{\text{divergence}}}{q_{\text{reversal}}}ight) \left(\frac{C_{t_0}}{C_{t_0}}\right)$$

and $q = \sqrt{\frac{q_{\text{divergence}}}{q_{\text{reversal}}}$. In general, $f$ is a negative number.

The hinge moment due to the aerodynamic pressure on the flap is written as $M_\delta = H_\delta \delta_o$ where $H_\delta = \int_{x_{uper}} p_\delta (x-x_H) dx$. The work done by the aileron is $W_{\text{aileron}} = \frac{\delta_o^2}{2}$. This work is zero if the hinge moment is zero. This special case occurs at airspeeds where the following is true.

$$\int_{x_{uper}} p_\delta (x-x_H) dx = -f \int_{x_{uper}} p_\theta (x-x_H) dx.$$  (2)

This occurs when $f = \left(\frac{-q}{1-q}\right) \left(\frac{C_{t_0}}{C_{t_0}} + c \frac{C_{max}}{C_{t_0}}\right) = \frac{\int_{x_{uper}} p_\delta (x-x_H) dx}{\int_{x_{uper}} p_\theta (x-x_H) dx} = \frac{-a_\delta}{a_o}$. The dynamic pressure at which this occurs is defined as $q_o$ and is found to be

$$q_o = \frac{q_{\text{divergence}}}{q_{\text{reversal}}} = \frac{-a_\delta}{a_o \left(\frac{C_{t_0}}{C_{t_0}} + c \frac{C_{max}}{C_{t_0}}\right)}$$  (3)
Plots of $\bar{q}$, and the nondimensionalized reversal dynamic pressure $\bar{q}_{\text{reversal}} = \frac{q_{\text{reversal}}}{\bar{q}_{\text{divergence}}}$ are presented in Figure 3, plotted as functions of the aileron hinge line location (or flap-to-chord-ratio) and nondimensional dynamic pressure $\bar{q} = \frac{q}{\bar{q}_{\text{divergence}}}$. Reversal occurs before the zero moment dynamic pressure occurs. Near this combination of dynamic pressure and hinge moment location, this airfoil/control combination requires only a small amount of work to generate lift. Theoretically, at $\bar{q}$, the aileron work required is zero because the airstream does all of the work.

The efficiency of the aileron in generating lift is measured as the lift generated per unit of work input into the aileron control surface. A contour plot of lift generated by the aileron, divided by the work done by the aileron, is provided in Figure 4. In this plot, the logarithm of the ratio is plotted. Figure 4 also shows lines of equal lift. Those nearest reversal are smallest because of lift ineffectiveness. Note that the most efficient flap-to-chord and dynamic pressure combinations are those near $\bar{q}$. The most inefficient operation occurs near reversal, the dark blue line in Figures 3 and 4.

Several conclusions may be drawn from this simple study. First of all, aeroelasticity matters and affects operation of the control device. Second, the stiffness of the structure and the size of the actuator can be tailored for low energy performance. A similar study with a leading edge device, not shown here, shows that there is a similar leveraging of aeroelastic effects for airfoil/structure design, although reversal does not occur.

**Wing actuator efficiency modeling**

The second problem to illustrate energy optimization for morphing wings uses the model shown in Figure 5. This wing model consists of two spanwise segments, each with a trailing edge or leading edge articulated surface. We want to find how to create a fixed amount of lift by deflecting control surfaces, but distribute the lift over the wing segments so that the total energy required to deflect the surfaces is minimized.

For models of this type, the articulated surface hinge moment, $M_i$, is related to the ensemble of control
deflections $\delta_i$ by the equation

$$M_i = \sum_{j=1}^{n} H_{ij} \delta_j$$  \hspace{1cm} (4)$$

where the coefficients $H_{ij}$ are hinge moment coefficients that provide the influence of each surface on the hinge moment in question. The expression for the ensemble of hinge moments is \{ $M_i$ \} = \{ $H_{ij}$ \} $\{ \delta_j \}$ so that the work done by the controllers is

$$J = \frac{1}{2} \{ \delta_j \} \{ M_i \} = \frac{1}{2} \{ \delta_j \} \{ H_{ij} \} \{ \delta_j \}$$  \hspace{1cm} (5)$$

We normalize this expression by dividing by a reference coefficient so that

$$J_o = \frac{1}{H_o} \left( \frac{1}{2} \{ \delta_j \} \{ M_i \} \right) = \frac{1}{2} \{ \delta_j \} \left[ \frac{H_{ij}}{H_o} \right] \{ \delta_j \}$$  \hspace{1cm} (6)$$

If we simplify our aerodynamic model to remove the cross-coupling terms $H_{12}$ and $H_{21}$ and assume that $H_{11}$ and $H_{22}$ are equal then the performance index $J_o$ simply becomes

$$J_o = \frac{1}{2} \delta_1^2 + \frac{1}{2} \delta_2^2$$  \hspace{1cm} (7)$$

The objective is to generate a fixed amount of lift $L_o$ with the control surfaces that deflect amounts $\delta_1$ and $\delta_2$. Figure 6 illustrates the fact that the energy minimization problem in which we minimize Eqn. 7 while generating a specified lift has a closed-form solution.

When we use simple aerodynamic strip theory, the equations of static equilibrium provide two simultaneous equations in terms of the twist angles of each section and torsional stiffnesses of the springs connecting the panels with each other and the wall. The requirement for constant lift $L_o$ plots as a straight line $L_o = A_1(q) \delta_1 + A_2(q) \delta_2$, as shown in Figure 6.

The coefficients $A_1$ and $A_2$ are functions of dynamic pressure, $q$, because of aeroelastic effects. The least energy solution is found by locating the tangent between the circle defined in Eqn. 7 and the constant lift line. When this tangency point is input into Eqn. 7 the result is

$$J_{opt} = \frac{1}{2} \frac{L_o^2}{A_1^2 + A_2^2}$$  \hspace{1cm} (8)$$

Since $A_1$ and $A_2$ depend on aeroelasticity, $J_o$ is a function of the aeroelastic features of the design. If, at a certain dynamic pressure, one of the control surfaces reverses, then the coefficient $A_i$ for that surface will be zero and the constant lift line in Figure 6 will be parallel to one of the axes. The minimum energy solution will require that only one of the surfaces generate lift while the other is idle.
Figure 7 shows how the aileron deflection on each panel changes with non-dimensional dynamic pressure \( \bar{q} = \frac{q}{q_{\text{divergence}}} \). The vertical axis measures the amount of aileron deflection (per radian) per unit of required wing lift. Note that the sum of the lift generated by each section is constant. As airspeed increases, if the aileron on one of the panels reverses, it will then deflect upward (a negative number in Figure 7) to create positive lift.

Figure 8 plots the non-dimensional energy function \( J_0 \) as a function of non-dimensional dynamic pressure \( \bar{q} = \frac{q}{q_{\text{divergence}}} \).

Figure 8 shows that the energy required is large between the reversal points of individual surfaces shown in Figure 7. The maximum amount of energy required does not occur exactly at the reversal point for either aileron. Instead, there is a point where the combination of structural stiffness and aileron arrangement creates the need for large actuator energy inputs to achieve the required lift. The dynamic pressure at which this occurs can be located exactly and this solution will be discussed in the next section.

**Multi-degree-of-freedom wing actuator model**

A more accurate, but still simplistic, model of a flexible wing with leading and trailing edge actuators provides further insight to the minimum energy actuation lift generation problem. The model shown in Figure 9 uses a distribution of horseshoe vortices on a flexible, beam-like wing to provide matrix equations for spanwise lift as a function of wing spanwise angle of attack distribution and control surface deflections. The static equilibrium of forces and moments along the wing reference axis is written in matrix form as

\[
\left[ \frac{1}{q} \bar{A}_y \right] \left[ \begin{array}{c} \beta_y \\ \beta_z \end{array} \right] = \left\{ \delta_z \right\} + q \left[ \begin{array}{c} \beta_y \\ \beta_z \end{array} \right] \left\{ \delta_y \right\}
\]

or

\[
\left[ B_y \right] \left\{ \beta_y \right\} = \left\{ \delta_y \right\} + q \left[ E_y \right] \left\{ \delta_y \right\}
\]

so that the lift distribution is

\[
\left\{ \beta_y \right\} = \left[ B_y \right]^{-1} \left\{ \delta_y \right\} + q \left[ E_y \right] \left\{ \delta_y \right\}
\]

\[
\left\{ \beta_z \right\} = \left[ B_z \right]^{-1} \left\{ \delta_z \right\} + q \left[ E_z \right] \left\{ \delta_z \right\}
\]
The matrices $[Z_y]$ and $[E_y]$ are functions of controller aerodynamic derivatives, location and wing bending and torsional stiffness. The matrix $[B_y] = \left[ \frac{1}{q} [A_y] - [C_y] \right]$ is the aeroelastic flexibility matrix for the symmetrical, cantilevered wing. This matrix is a function of the distribution of aerodynamic derivatives along the wing, either estimated or measured, as well as planform geometry and structural stiffness. The distribution of known initial angles of attack along the wing is given by $\{\alpha_i\}$ while the distribution of control surface deflections is given by $\{\delta_i\}$. For this study, we set $\{\alpha_i\} = 0$ so that

$$\{l_i\} = [B_y]^{-1} \left( [Z_y] + q[E_y] \right) \{\delta_i\}$$

(12)

If the design objective is to use active control surfaces to increase lift (to any distribution favored by the designer), then we can use combinations of full span ailerons or full span leading edge devices to do this. If there are as many control elements as there are panel segments then there is a closed-form solution to this problem. For instance, if we wish to compute the required aileron deflections to create a lift distribution with known resultant and an elliptical distribution for minimum drag, then we can compute the distribution to be

$$\{\delta_i\} = [Z_y] + q[E_y]^{-1} [B_y] \{l_i\}_{\text{elliptical}}$$

(13)

In this case, the solution is closed-form and does not require minimization because there are no free design variables.

Equation 13 requires the inversion of the matrix $[Z_y] + q[E_y]$. This matrix is a function of the wing torsional and bending stiffness, as well as the size and location of the leading edge and trailing edge devices. This suggests the following eigenvalue problem

$$[Z_y] + q[E_y] \{\delta_i\} = 0$$

(14)

or

$$[E_y]^{-1} [Z_y] \{\delta_i\} = -q \{\delta_i\}$$

(15)

Equation 15 shows that there is a self-equilibrating deflection shape $\{\delta_i\}$ at the dynamic pressure, $q_0$, for which the deflected surfaces and wing distortion combinations are in static equilibrium no matter what the magnitude of the deflection. However, at this airspeed the ailerons cannot create lift. This is a multi-degree of freedom reversal speed, but it differs from the classical reversal speed, since the classical divergence speed is for a single aileron instead of an ensemble of ailerons.

Equation 15 is an eigenvalue problem and the control deflections are an eigenvector. Near or at the special eigenvalue $q_0$, the ailerons will generate lift if they are not deflected in the eigenvector shape, but there is a high actuator energy cost associated with this control-induced lift. To study this phenomenon, we created a constant chord, swept wing model representative of a high-altitude flying wing.
The wing sweep angle is 25.5° while the span is nearly 200 feet. The equations for static equilibrium given in Eqn. 9 were programmed into a MATLAB source code. The optimization problem is to minimize $J_0$ subject to the constraint that $L_0 = 4000$ lb. This problem was solved using the MATLAB optimization tool kit. This configuration was found to have a reversal eigenvalue at $M=0.38$.

Figure 10 shows the solution for minimum energy trailing edge flap deflections (in radians) as a function of Mach number. Depending on their location along the span, the outboard control surfaces begin to reverse at relatively low Mach numbers (indicated by the change in deflections from positive to negative in Figure 10). The inboard panels do not reverse in the Mach number range considered.

Figure 11 shows how the lift distribution adjusts as Mach number increases. As Mach number increases, the inboard panels (on the left side in this figure) must generate more of the load as the outboard ailerons begin to reverse. As Mach number increases, the reversed trailing edge surfaces become more effective in their reversed state than the inboard panels, causing the majority of the load to shift outboard. The spanwise lift distribution is distorted and induced drag changes. Figure 11 indicates that there is a dramatic increase in the inboard lift at about $M=0.38$, followed by an increase in the outboard lift as the reversed panels begin to generate more load.

Figure 12 shows the minimum actuation energy function $J_0$ as a function of Mach number. In contrast to the 2-degree-of-freedom model, there is no peak in actuation energy, but instead there is a steady decline in the energy due to aeroelastic effects, in this case the operation of reversed aileron segments.

As presently posed, there is no constraint on spanwise lift distribution, only a requirement that the lift have a specified net value. As a result, the ailerons are free to avoid actuator shapes that resemble the eigenvector at our reversal eigenvalue of 0.38. If we add an additional constraint to the problem and force the spanwise lift distribution to be elliptical so
that we include minimum induced drag into the problem, the results are very different. Because the number of design variables (deflections) equals the number of constraints (elliptical panel lift distribution with a specified net value), there is only one combination of trailing edge deflections that will produce an elliptical lift distribution. Therefore, energy is not minimized, it is simply calculated and the formal optimization problem disappears.

Figure 13 shows the trailing edge surface deflections required to create an elliptical lift distribution. The deflections become large at the critical reversal Mach number \( q_o \) defined in Eqn. 15.

Figure 14 shows the control energy required to generate an elliptical lift distribution. As the determinant of the matrix \( \begin{bmatrix} Z_{\psi} + q_0 E_{\psi} \end{bmatrix} \) becomes small and the trailing edge deflections become large and the actuation energy function also becomes large, but not infinite since the control surface deflection pattern is not exactly the same shape as a system eigenvalue.

When both leading edge and trailing edge surfaces are included in the control problem with elliptical lift distribution requirements, an optimization problem once again appears since there are more design variables than there are constraints. Figure 15 shows the leading and trailing edge deflections that produce an elliptical lift distribution with minimum control energy. At the critical reversal Mach number, the trailing edge surfaces are unable to produce the required distribution without substantial energy expenditure so the more effective leading edge surfaces begin to be deployed.

Figure 16 shows the minimum actuation energy function \( J_o \) required to create an elliptical lift distribution with a combination of leading and trailing edge surfaces. The energy does have a peak associated with the system eigenvalue \( q_o \) but it no longer is extremely large. The addition of leading edge flaps allows the lift distribution to be maintained with reasonable control surface deflections.

**Summary**

These reversal points and the associated reversal problems are revealed by an eigenvalue problem such as that discussed in this section. For trailing edge surfaces, this eigenvalue problem will most likely define regions of ineffectiveness that are within the flight envelope. These regions are functions of the usual ensemble of aeroelastic parameters, including torsional and bending stiffness and flap-to-chord ratio.
Leading edge surfaces have eigenvalues that are usually negative and thus cannot experience reversal. When a combination of leading edge and trailing edge surfaces are used and a formal optimization routine is employed, effective combinations of lift generating surfaces are revealed.

**Minimizing strain energy associated with airfoil shape change**

Modern morphing wing concepts contain segments with internal controllers that continuously change the shape of the wing in flight without the hinge-line discontinuity associated with articulated surfaces (cf. References 4-9). In such cases, the efficient design of morphing structures to create lift and moments in flight must consider the structural strain energy required to change the lift on the surface as it transitions from one airfoil shape to another. The final part of our study considered how to deform two dimensional airfoil sections using the least amount of energy to achieve the goal of creating lift. The stated goal is to determine how to pose a useful, formal optimization problem to help us in the design process and actuator selection process.

When an airfoil changes its shape, the strain energy change required depends on the type of sub-structure enclosed within the airfoil shape. This internal topology must be designed, together with the actuators and their locations. To examine how much energy is required to change from one airfoil shape to another, we assumed that the airfoil structures are solid, cellular structures, idealized as a continuous collection of internal springs. Aerodynamic and aeroelastic effects were not considered for this study.

The airfoil shapes used in the study were limited to the NACA 4-series\(^{10}\) to ensure that all of the shapes were smooth and realizable. These airfoils have numbers such as NACA 2412. This airfoil series is described by three design parameters: the first number is the maximum camber-to-chord ratio, 2 being 2%, the chordwise location of maximum camber, 4 being the 40% position, and the thickness-to-chord ratio, 12 being 12% thick. All airfoils considered for this study have the maximum camber location fixed at the 40% chord position. Any candidate airfoil shape is described by only the camber-to-chord and thickness-to-chord design variables.

Because no aerodynamic pressures are included in our model, the only expenditure of energy is the strain energy change required to transition from one airfoil shape to another. Choosing an initial airfoil shape allows construction of an energy response surface to describe the strain energy associated with shape changes. For this study, the maximum camber varied between 0% and 9% of the chord and the thickness-to-chord ratio varied from 0% to 25%.

Strain energy is developed in the springs when force actuators displace the initial airfoil contour to generate a new shape specified by new maximum camber and thickness values. There are two springs at each chordwise location. One spring connects the camber line to the upper surface while the other spring connects the camber line to the lower surface.

The strain energy associated with the change in spring lengths is given as:

\[
U = \sum_{i=1}^{n} \frac{1}{2} k_i \Delta L_i^2 \quad (16)
\]

where \(k_i = \frac{EA}{L_i}\) is the stiffness of spring \(i\) and \(\Delta L_i\) is the change in length of spring \(i\). Equation 16 becomes

\[
U = \frac{1}{2} \sum_{i=1}^{n} \frac{EA}{L_i} \Delta L_i^2 \quad (17)
\]

Equation 17 can be non-dimensionalized with respect to the chord length, \(c\), to give

\[
U_{N.D.} = \frac{U}{EAc} = \sum_{i=1}^{n} \frac{1}{2} \left( \frac{\Delta L_i}{c} \right)^2 \quad (18)
\]
During airfoil deformation, the camber line of the original airfoil remains grounded, so that the change in spring length is governed solely by the displacement of the airfoil’s outer surfaces with respect to the initial camber line. This modeling approach is illustrated in Figures 17a and 17b. This modeling scheme reflects a shape change created by actuators applying vertical forces.

By choosing an initial airfoil shape with a given lift coefficient and displacing it into all other allowable shapes, it is possible to determine the strain energy response surface. Figure 18 shows this response surface using the NACA 2412 airfoil as the initial, reference airfoil. The reference strain energy is zero. The constant strain contours are shown as bold lines. Figure 18 also shows lines of constant $c_l$ (thin solid lines) and constant $c_d$ (thin dashed lines). Lines with larger $c_l$ values are located nearer the top of Figure 18 while larger drag values are located in the upper right part of the figure.

Figure 18 shows that there are many airfoil shapes capable of satisfying a constraint on $c_l$, but there exists only one that can be modified for the least amount of energy expended to deform the surface. If constraints exist on both $c_l$ and $c_d$ there is no optimization problem because only one airfoil shape satisfies both of the constraints. However, since the aerodynamic performance constraints are predetermined, results such as those in Figure 18 can help to determine what these constraints should be by showing the tradeoffs between aerodynamic performance and energy. For instance, it might be seen that for a small increase in allowable drag, large reductions in actuation strain energy can be obtained.

**Adaptive shell models**

The internal spring model strain energy is based on a cellular wing structure with controlled displacements from the original camber line. However, the linear spring model does not explicitly describe the strain energy associated with a shell structure like that usually used in wing design. In particular, the shell skin may undergo substantial perimeter changes as the airfoil shape changes. Figure 19 shows a
representative finite element model created using ASTROS\textsuperscript{11} (Automated STRuctural Optimization System).

There are two ways to change from one airfoil shape to another. We can simply move points straight upward from the initial position or we can move points at an angle. Although the final shape is the same, the amount of bending and stretching of the shell is different. In both cases the changes in the airfoil shape were created by defining the nodal displacements of the final airfoil shape. When we move the initial points vertically, the strain energy distribution is shown in Figure 20. Note that strain energy is concentrated at the leading edge because leading edge elements have large bending and in-plane strains, while mid-chord elements undergo nearly rigid-body translations.

When nodes are displaced along a vector that yields the shortest distance between the initial and final airfoil shapes the strain energy distribution is shown in Figure 21. These results indicate that this alternative displacement method distributes the strain energy over a larger area of the airfoil and the leading edge stress concentration is not as large.

![Figure 21: Typical strain energy distribution resulting from the shape change of a 2412 airfoil via minimum distance nodal displacements](image)

When the shell model is compared to the original cellular structure model the shapes of the strain energy response surfaces are different. There are also differences between the two nodal displacement methods. These differences are shown in Figures 22a-c. These results imply that the strain energy developed during a shape change depends not only on the initial and final airfoil shapes, but also on the manner in which the shape change is carried out. They also imply that one type of actuator may be better than another so that the shape change has minimum actuation energy. Optimization to minimize the strain energy associated with an airfoil shape change is dependent upon the actuation approach used to create the shape change.

**Conclusion - actuator/structure integration and aeroelasticity matter**

The development of smart materials and new actuators has created the ability to design and use continuously deforming surfaces to generate aerodynamic forces and moments. Aeroelastic effects allow the airstream to do work on the airfoil to generate lift with reduced actuator effort. As a result, there are flight regions where certain types of actuators operate more efficiently. However, continuously deforming surfaces encounter different problems than conventional surfaces because they may be deployed in an infinite number of different shapes and some deployment schemes are better or more effective.

Continuously deforming control surfaces are subject to a special reversal phenomena that limit their effectiveness. These reversal problems are revealed by an eigenvalue problem that has a critical dynamic pressure and a mode shape. The result is a range of Mach numbers or dynamic pressures where the surfaces are ineffective. For trailing edge surfaces, these ineffectiveness regions are functions of an ensemble of aeroelastic parameters, including torsional and bending stiffness and flap-to-chord ratio.
Continuous leading edge surfaces have critical reversal eigenvalues or dynamic pressures that are negative; these surfaces do not experience control reversal. When a combination of leading edge and trailing edge surfaces are used and a formal optimization routine is employed, effective combinations of lift generating surfaces are revealed.

When the surface itself is deformed by internal actuators that strain the structure, some shapes that generate the required lift are easier to morph into. The results shown here indicate that strain energy is a useful metric to use to judge how to go from one lift requirement to another. What is needed is a model with an optimization procedure that, unlike today’s weight minimization, is multi-disciplinary in that it considers metrics such as strain energy as the performance index and takes into account changes in the structure and aerodynamic measures such as lift and drag.

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