A Design and Analysis of a Morphing Hyper-Elliptic Cambered Span (HECS) Wing

Leonard D. Wiggins*, Matthew D. Stubbs†, Christopher O. Johnston‡, Harry H. Robertshaw*, Charles F. Reinoltz†, and Daniel J. Inman#

Center for Intelligent Material Systems and Structures
Department of Mechanical Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia  24061

The HECS wing, developed by NASA Langley Research Center, features a nonplanar, hyper-elliptically swept leading and trailing edge as well as spanwise camber. In this paper, we propose a single-degree-of-freedom mechanism to provide a means for the wing to continuously morph from a planar to a nonplanar configuration. The mechanism, which is something like a scissor linkage, uses a repeating quaternary-binary link configuration to translate the motion from one wing segment to the next. The mechanism is synthesized such that, with one input to the first segment in the chain, the other wing segments move into their desired positions. To predict the aerodynamic loads associated with this morphing dihedral change, linear theory is applied to the HECS wing configuration at distinct morphed positions. For the structural study, a finite element representation of the mechanism is developed, and a linear static analysis at different morphed positions is performed. Using the predicted aerodynamic loads, the structural analysis investigates different materials and cross sections of the mechanism members to determine a need for redesign due to failure from buckling and bending stress. A design is finalized which, compared to the design of the original model, lightens the structure as well as increases its strength. These results are beneficial for the next phase of model development of the mechanism. This work shows that a relatively simple kinematic mechanism can produce the desired range of motion for a variable dihedral HECS wing. It also provides insight into the aerodynamic effects of the nonplanar wing configuration with an analysis of the structural integrity of the mechanism under loading.

Nomenclature

\begin{align*}
i & = \text{ Unmorphed position} \\
j & = \text{ Morphed position} \\
A & = \text{ Member attachment point} \\
B & = \text{ Member attachment point} \\
D_i & = \text{ Induced drag} \\
L & = \text{ Lift} \\
q & = \text{ Dynamic pressure} \\
b & = \text{ Projected wing span} \\
e & = \text{ Span efficiency factor} \\
\sigma_{\text{cr}} & = \text{ Critical force} \\
E & = \text{ Young's modulus} \\
I & = \text{ Area moment of inertia} \\
L_m & = \text{ Member length} \\
S_r & = \text{ Slenderness ratio} \\
k & = \text{ Radius of gyration} \\
\sigma_{\text{max}} & = \text{ Maximum bending stress}
\end{align*}

I. Introduction

Research and development within the field of morphing aircraft technology deals with the continuous change in wing shape, unlike conventionally actuated wings which use discrete control surfaces such as hinged flaps. Some methods of wing morphing\textsuperscript{1,2,3,4} that have been previously studied include variable twist, camber, sweep, dihedral, and span. The HECS wing design has a hyper-elliptically swept planform in addition to a nonplanar hyper-elliptical cambered span. To change the HECS wing configuration between a nonplanar and
planar wing, a variable dihedral type of planform morphing must be implemented. Figure 1 shows both a planform and span view of the nonplanar and planar positions which are to be achieved by morphing.

![planform view](image1)

(a) Planform view

![span view](image2)

(b) Span view

Figure 1. An illustration of the final nonplanar and planar positions of the morphing HECS wing

Several multiple-degree-of-freedom mechanism concepts have been proposed as candidates to morph the variable dihedral wing in this manner. Tensegrity structures, variable geometry trusses, compliant mechanisms, or other so-called smart structures have been considered. These devices generally have many degrees of freedom and thus require many inputs. Because of this and other complexities, these designs can be heavy, and difficult to build and control. So far, there has been little or no research proposing the synthesis of a single-degree-of-freedom linkage in the design of morphing dihedral wings. Our research has been focused on the design of a practical, single-input mechanism that can move the HECS wing between its nonplanar and planar configurations.

The motivation for variable dihedral capability comes from two conflicting design objectives: an aircraft that achieves both efficient cruise and high maneuverability. For low speed efficient cruise, the main objective is to minimize the induced drag. This leads to a wing with the largest span possible, or in the case of the HECS wing, the planar or undeflected wing position. For maneuvering at low lift coefficients, it is desired to minimize the wing-root bending moment and maximize the obtainable roll-rate. The nonplanar fully deflected wing achieves this objective and also greatly improves the lateral-directional stability. The variable dihedral capability can also be used for longitudinal and lateral-directional control.

A morphing dihedral wing could also be applied to wing-in-ground-effect (WIG) vehicles. While WIGs fly close to the surface, they profit from the increase in lift and reduction of drag. However, when these vehicles fly out of ground effect, they become inefficient with altered stability. A variable dihedral aircraft, in its full dihedral configuration, can achieve the benefits of reduced drag and increased lift while in ground effect. Yet, by morphing to planar wing configuration, it can still be efficient and controllable at higher altitudes.

The next section covers the development of the single-degree-of-freedom morphing mechanism. Then, a description of the aerodynamic analysis of the wing is covered, followed by an investigation into the structural analysis of the mechanism. The paper ends with conclusions and acknowledgements.

II. Kinematic Model Development

The kinematic mechanism developed consists of a serial chain of single-degree-of-freedom, single-input, four-bar mechanisms, as illustrated in Figure 2. The chain is formed from a set of quaternary (four joint) links that provide the main structure of the wing, along with binary (two joint) links that serve to actuate the quaternary links. The mechanism from this point forward will be called a quaternary-binary cross linked mechanism (QBCLM). The first section of the chain acts as the input to the following sections through a set of binary links. Rotating the quaternary link of the mechanism causes every other quaternary link to rotate a pre-specified amount. To produce the desired motion, the dimensions of the linkage must be determined through kinematic synthesis.

![model developed](image3)

Figure 2. An illustration of the model developed. Possible actuation point labeled with arrow.

Airfoil rib sections are to be mounted on the quaternary links. Since the quaternary links form the main wing structure, the mechanism approximates the nonplanar shape in a piece-wise linear manner according to the number of quaternary links. The desired rib sections in their planar and nonplanar positions are shown by triangles in Figure 3. The chosen distance between rib locations defines one dimension of the quaternary links, called the span of the section. It remains to choose the locations of the binary link attachment points at each end, yielding four design variables for each section. Novel synthesis techniques were developed to locate the attachment points, yielding a mechanism which meets the shape requirements.
A. Mechanism synthesis

A synthesis procedure was developed with the goal of matching the dihedral shapes shown in Figure 3 with a piecewise linear curve. The required span (S) of each section was determined directly from this figure, while the other necessary dimensions of the QBCL mechanism were synthesized to meet the required shape. The four design variables consist of the y and z locations of points A, and the attachment points of the binary link, B. Each four-bar section of the wing can be synthesized separately. The representation shown in Figure 4 is suitable for any two adjacent sections of the wing. In Figure 4, CO and CA represent the defined chord sections from Figure 3, while CB represents the location of structure for attachment of the binary link. The angle θA and θB define the desired morphed configuration of this segment.

\[ y_{bi} = S + Y_{RB} \]
\[ z_{bi} = Z_{RB} \] (1)

However, in the in the fully deflected (j) position these components become,

\[ y_{bj} = S \cos(\theta_A) + Y_{RB} \cos(\theta_t) - Z_{RB} \sin(\theta_t) \]
\[ z_{bj} = S \sin(\theta_A) + Y_{RB} \sin(\theta_t) + Z_{RB} \cos(\theta_t) \] (2)

where \( \theta_t = \theta_A + \theta_B \).

If the links are considered rigid, the distance between A and B must remain constant throughout the motion of the mechanism. If L represents the distance between A and B, then \( L_i \) must be equal to \( L_j \),

\[ L_i = \sqrt{(y_{ai} - y_{aj})^2 + (z_{ai} - z_{aj})^2} \]
\[ L_j = \sqrt{(y_{bj} - y_{aj})^2 + (z_{bj} - z_{aj})^2} \] (3)

Although there are many solutions for the points A and B, important constraints exist that govern which solutions may be chosen. The size of the mechanism is constrained to fit within the wing geometry. Excessively long, slender mechanisms are generally not desired. The synthesis code was written with these parameters in mind. Points A and B are chosen to lie at the most extreme positions relative to each other, in an attempt to generate the most desirable mechanism with regard to structure.

The third section of this mechanism is taken as an example of how the attachment points are chosen. The most extreme position for B is at the \((y, z)\) location of \((S, C_A/2)\). Locating B removes two free choices and a single free choice for the location of A is left. The locus of possible attachment points \(A_n\) is found using a numerical root finding tool, and is shown in Figure 5. Point A is chosen to lie directly above O, yielding a fully synthesized section to match the shape forming requirements. Every section is synthesized in this manner to find all \(A_n\) and \(B_n\).

B. Note on Body Guidance

The unique manner in which the synthesis methods were developed is due to the physical nature of the problem. In the current design phase, airfoil chord sections must be placed at specific locations along the span of the wing. This led to the notation shown in Figure 4. For those familiar with kinematic synthesis techniques, this is equivalent to a classic function generation problem, where the angle of each quaternary link is specified in two positions.10
C. Kinematic Results

A physical model of the mechanism was developed based on the synthesized design. A simple skeleton model of the first four sections of the morphing wing mechanism, in an intermediate position, is shown in Figures 6 and 7 to prove the concept. This is a first generation model which did not include sweep. A second generation model was constructed that includes the sweep. Shown in Figures 8 and 9, the model also uses metal brackets and threaded rods for the binary links.

III. Aerodynamic Analysis

The aerodynamic loads are calculated using a nonplanar vortex lattice method. This is a linear theory that assumes an undeformed wake. The validity of assuming an undeformed wake is questionable, but nevertheless, this assumption has been shown to only slightly affect the aerodynamic forces. Three example dihedral shapes are shown in Figure 10. Figures 11 and 12 show the section lift coefficient \( C_l \) and side force coefficient \( C_s \) distributions for the wing shapes shown in Figure 10. The increase in \( C_s \) towards the wing tip for the deflected cases is to be expected because the aerodynamic (pressure) forces act normal to the wing surface. Examination of Figures 11 and 12 show that the distribution of lift and side force produced by angle of attack and airfoil camber are different. The forces caused by airfoil camber (from an SD 7032 airfoil) are concentrated closer to the wing tips than the forces due to angle of attack. This is a result of the reduced local angle of attack of the deflected wing sections near the wing tip, whereas the effectiveness of camber is unaffected by the local dihedral.

The induced drag characteristics of nonplanar wings have been studied by Cone, Lowson, and Smith and Kroo. They show that although the span efficiency factor \( e \) increases for nonplanar wings, the total induced drag also increases if the structural span (arc length) is held constant. This result is seen through the following equation for the induced drag,

\[
D_i = \frac{1}{q} \frac{L^2}{\pi e b^2} \quad (4)
\]

where \( L \) is the lift, \( q \) is the dynamic pressure, and \( b \) is the projected wing span (projected onto the \( y \)-axis). Figure 10 shows that the projected span decreases by 11% when going from the flat case to the fully deflected case. Using Blackwell’s method, it was found that \( e \) increases by 15% when going from the flat case to the fully deflected case. When substituting these values into Equation 4, it is found that for a given lift, the drag for the fully deflected case is about 10% greater than that for the flat case. For the drag to have decreased in this case, the value for \( e \) would have to have been at least 1.26 for the fully deflected wing. This indicates that although it is easy to increase \( e \) above 1 for a non-planar wing, it is difficult to constrain the loss of projected span so that an actual decrease in induced drag is achieved (over a planar wing of the same structural span). It should be noted that wing sweep does not affect the minimum induced drag predicted by linear theory. Possible nonlinear drag benefits of the aft-swept tips of the HECS wing are discussed by van Dam, Smith and Kroo, and Burkett.
Further discussions on the drag of nonplanar wings are given by Mortara and Maughmer and Kroo.

**Figure 10.** Example wing dihedral shapes and illustration of the section lift and section side-force coefficients.

**Figure 11.** A plot of the aerodynamic load distribution per-degree angle of attack for different wing shapes.

**Figure 12.** A plot of the aerodynamic load distribution due to airfoil camber (SD 7032 airfoil).

### IV. Structural Analysis

With the kinematic model and aerodynamic study established, an analytical 2-D structural study of the mechanism is performed. A successive approach, detailed by Rivello, is used for the structural analysis of the mechanism. For this successive approach, the applied loads are determined, a tentative design is established, the actual stresses and deformations are found, and the allowable stresses and deformations are calculated. Based on certain criteria, the design is determined to be adequate by evaluating the actual and allowable stresses and deformations. There are two categories for the 2-D study. For one category, the gravitational force study, a strength-to-weight factor is used as the criterion to validate the design with a goal of reducing the static deflection. The other category is an aerodynamic and gravitational force analysis. Buckling margins of safety and factors of safety are used as the criteria to determine if the design is adequate with the goal of reducing the buckling and maximum bending stress of the mechanism members.

The kinematic design is represented with a 2-D finite element model in ANSYS. When modeling in ANSYS, the second generation design concept is taken into consideration for the tentative cross section and material of the structural components. The binary links of the mechanism are modeled as LINK1 elements, which are 2-D spar truss elements. There are two translational degrees-of-freedom per node with these elements. The quaternary links are modeled with BEAM3 elements. These elements have uniaxial tension, compression and bending with three degrees-of-freedom, two translational and a rotational per node, which is an accurate representation of the concept model. The metal brackets are also modeled as BEAM3 elements. These brackets experience tension and compression as well as bending moments caused by the binary links and the applied forces, making the beam elements a good representation.

Three real constant sets are also defined in ANSYS, one for each of the three element types. These constants define parameters such as cross sectional area, area moment of inertia, and beam height. The binary links have circular cross sections, and the quaternary and bracket links have rectangular cross sections.

When creating the geometry of the structure, nodes are created at the defined revolute joint locations. Elements are created from these nodes, and the elements are assigned attributes according to its element type. For example, when the binary links are created, it is made certain that they use the corresponding LINK1 elements, real constant sets, circular cross sections, and material models. The same logic applies to the quaternary and bracket links. Fixed boundary conditions are
applied to nodes at the wing root of the structure. The gravitational loads mentioned earlier are simulated through the ACEL command of ANSYS. Through that command, the densities of the materials are taken into account to create defined gravity loads.

The distributed loads from the aerodynamic analysis are used as the applied loads on the structure. Load distributions in the flat, fully deflected, and half deflected positions are determined from the aerodynamic study. The flat case is used as the conservative worst case position since it creates the largest bending moment on the wing. Numerical integration of an area surrounding the rib locations gives concentrated forces which are used as the applied loads through the revolute joint nodes. Figure 13 illustrates the structure as it is modeled in ANSYS.

An investigation is performed on the flat position of the morphed HECS wing since this case produces the most extreme bending moment on the root of the wing. Two basic structural analyses are carried out on the mechanism in this flat position. One analysis investigates the effects of the weight of the structure by using a gravitational study. The other analysis uses the predicted aerodynamic loads as well as the structural weight of the mechanism. Both of the analyses follow the successive or iterative type approach. Also note that the binary links are labeled from B1 to B10 and the quaternary links from Q1 to Q10 from root to tip, respectively.

### A. Gravitational Force Study

ANSYS is used to determine the stress in the members and the deflection of the structure. Calculations are performed to determine the strength-to-weight ratio factor used to establish if the structure should be redesigned according to its deflection. In order to find the strength-to-weight ratio, the ultimate tensile strength of the material is divided by its weight density.

When the model is solved to investigate gravity effects for the initial design, a maximum displacement of 2.12 in. occurs at the tip of the structure. The materials and corresponding strength-to-weight factors of the links are shown in Table 1. The maximum tensile stress occurs in the first link as expected. The maximum value of 952.1 psi does not approach the ultimate tensile strength of 1.15E5 psi of the plain carbon and low alloy steel material in which it is modeled with. This shows that the binary links are not approaching failure in tension from the weight of the structure.

<table>
<thead>
<tr>
<th>Link</th>
<th>Material</th>
<th>Strength-to-weight (in^2/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary</td>
<td>Plain carbon and low alloy steel</td>
<td>1.57E8</td>
</tr>
<tr>
<td>Quaternary</td>
<td>Soft wood</td>
<td>8.07E6</td>
</tr>
<tr>
<td>Ground</td>
<td>Soft wood</td>
<td>8.07E6</td>
</tr>
<tr>
<td>Bracket</td>
<td>Plain carbon and low alloy steel</td>
<td>1.57E8</td>
</tr>
</tbody>
</table>

In order to decrease the maximum displacement due to gravity, more exploration is carried out using the finite element model. As noted, only an investigation of different materials is used for the gravitational study. The cross sectional areas of the links remain the same. The general trend follows that as the strength-to-weight ratio increases for the chosen material, the resulting maximum displacement of the structure decreases. A design choice of Aluminum Alloy 7075 T651 is made for the binary, quaternary, and ground links. The bracket uses the same plain carbon and low alloy steel material of the first test run. The resulting maximum stress occurs in the first binary link member. The maximum displacement of the structure is reduced from 2.12 in. of the initial design to 1.22 in. for the completed gravitational analysis design. Figure 14 shows an illustration of the deflection of the structure where the maximum displacement occurs at the tip. The design with the chosen materials leads to the aerodynamic and gravitational loads analysis.
Figure 14. ANSYS plot of nodal displacement results from completed design of the gravitational analysis of the flat case.

B. Aerodynamic and Gravitational Force Analysis

There are two phases of the aerodynamic and gravitational load analysis. One phase focuses on reducing the buckling margin of safety in the binary links. The other phase attempts to reduce the maximum bending stress of the quaternary links. As in the gravitational analysis, materials, real constants, and cross sections are set in the ANSYS model for the binary, quaternary, bracket, and ground links. ANSYS is used to determine the axial stress, bending moment in the members and the deflection of the structure. The results and designs of the buckling and bending moment phases are covered.

1. Buckling Analysis Phase

For the buckling analysis phase, the predicted aerodynamic loads are applied to the structure and buckling is investigated starting from the chosen design of the gravitational analysis. There is a potential for buckling with the two-force binary members because of the applied load. As discovered, the way in which the members are analyzed for buckling depends on the length of the link. The binary links are identified as B1 through B10 from root to tip the same as in the gravitational analysis.

For the analysis, the limit loads are the largest anticipated loads the structure will experience during its lifetime. It is impractical to set the loads at a level in which the structure will never fail, because the design will be inefficient from a weight standpoint. Therefore, the limit loads are set at a level which results in an acceptable low level of failure. A safety factor of 1.5 is used for inhabited aircraft. The ultimate load is defined as the limit load multiplied by the safety factor.

A buckling margin of safety is used to examine the design effectiveness of each test run. The margin of safety is calculated by dividing the critical stress by the limit stress and subtracting one. The limit stress is found by using the member stress determined through the ANSYS model as an ultimate stress. This ultimate stress is divided by the safety factor of 1.5 in order to have the limit stress. The critical stress is found by dividing the critical force by the cross sectional area of the binary link. The formula for the critical force is,

\[
P_{cr} = \frac{\pi^2 EI}{L_m^2}
\]

where \(E\) is the Young’s modulus, \(I\) is the area moment of inertia, and \(L_m\) is the member length. The goal of this phase of analysis is to decrease the margin of safety in the binary links. There must be a positive margin of safety, but lightweight structures are designed so that the margin is as small as possible.

For the initial design and first test run, it is found that the binary link with the highest axial stress is the first member as expected. It also has the lowest buckling margin of safety of 12.5. This first design has a constant radius for each binary link and a maximum displacement of 15.75 in. at the tip. The remaining test runs investigate using varying radii for the members. This provides an opportunity to decrease the margin of safety for not just the first link, but the others as well, making the structure more lightweight. An examination of the slenderness ratios of the final two links shows that they are short columns and can not be examined for buckling like the other long links. A short column has a slenderness ratio less than 10. The slenderness ratio is defined as,

\[
S_r = \frac{l}{k}
\]

where \(l\) is the length of the member and \(k\) is the radius of gyration. The equation for the radius of gyration is,

\[
k = \sqrt{\frac{I}{A}}
\]

where \(I\) is the area moment of inertia of the cross section and \(A\) is the cross sectional area. The final two binary links have slenderness ratios of 9.15 and 6.60, respectively. For these links, the yield strength of the material versus the axial stress is used as the determining factor.

For the final design of the buckling study, the cross sections of the binary links are designed to withstand buckling and they are tailored to the sections of the wing which do not experience high stress in order to reduce weight. It is found that the member with the highest stress and major influence on the displacement...
of the structure is B1. The final design of the buckling phase has a deflection of 14.195 in. compared to 15.75 in. from the initial design. The final design proves to stiffen the structure as well, and this design is used as the initial design of the bending stress phase.

2. Bending Stress Analysis Phase

Starting from the final design of the buckling phase, a reduction of the maximum bending stress of the quaternary links is examined. Modeled as beams, the bending stress occurs on the underneath side as the resulting applied load bends the beam upward. This is the side which is put into tension by the bending of the beam. Throughout the analysis for the bending stress, the buckling of the binary links is still examined in order to determine how they react to the design of the beam sections. The quaternary links are labeled Q1 through Q11 from root to tip. There is one more quaternary link than binary link because of the final section at the tip of the wing which is modeled as a quaternary link.

A factor of safety criterion is used to evaluate the designs of each run in the bending stress phase. The factor of safety is found by dividing the yield strength of the material by the actual bending stress. The yield strength of Aluminum Alloy 7075 T651 is 73,200 psi. The actual bending stresses are found through the ANSYS solution. The solutions are chosen ANSYS element table results from the negative side of each beam element since the beam is in tension when bending upward. A factor of safety of 1.5 is usually the standard for inhabited aircraft. It is a lower factor of safety than that used by other civil or machine structures, but they require extensive analysis and tests. This lower factor of safety also helps in the design of more lightweight structures needed for aircraft. The goal of the tests is to approach a factor of safety around 1.5 and to decrease the maximum displacement of the structure.

All cross sections of the quaternary links of the structure are modeled the same for the first test run. The section which has the highest bending moment and stress is link Q1. It has a bending moment of 12,087 lb-in, an actual bending stress of 36,851 psi, and a calculated maximum bending stress of 36,836.6 psi. The first section also has the lowest factor of safety of about 1.99, and the factors increase from the root to tip of the structure. The maximum displacement with this design is 14.2 in.

A change in the cross sectional height is investigated in order to decrease the factors of safety of the members following the first section and to decrease the maximum displacement of the structure. It is chosen to investigate the use of different heights from root to tip of the structure. This serves as a way to reduce some of the weight of the structure towards the tip where the loads are not as high. When choosing heights for the sections, maximum heights of the links are determined. These heights are based on a designed “box” in which the structure can still fit within the structure of the wing. The importance of this “box” is that it gives a maximum height range to work in.

The results of the test runs of the bending stress analysis show that the beam sections can be tailored by cross sectional height in order to reduce weight and bending stress. It is determined, much like the results of the buckling phase analysis, that the first member experiences the highest bending stress and has the lowest factor of safety. It also shows that in order to decrease the maximum displacement of the structure, the height of the first quaternary link has to be increased. By increasing this height, the factor of safety for the first link is also increased. To allow for a decrease in the maximum displacement at the tip of the structure, a higher factor of safety is allowed. Since the first link experiences the lowest factor of safety, it is the focus of designing based on the factor of safety.

A design which increases the heights of sections Q1 through Q8 and reduces the heights of sections Q9 through Q11 from the height used in the initial design is determined. The tailoring of heights gives a lighter structure towards the tip, while decreasing the bending stress near the root. The lowest factor of safety of 9.37 occurs at the first quaternary link. This demonstrates that the structure will not fail from the bending stresses. A maximum displacement result of 2.2 in. at the tip of the structure also results from this design. The design can withstand the maximum bending stresses and a reduction in maximum displacement from the original design is found.

Compared to the original design, with the final design, the actual bending moment is reduced from 12,087 lb-in to 10,258 lb-in and the actual bending stress is reduced from 36,851 psi to 7,815.7 psi. This is achieved through the tailoring of sectional heights. As a consequence to the added weight of the structure, the buckling margins of safety of the first and second binary links increase. The margin values are higher than the initial, but are reasonable compared to the increased structural support and factors of safety. A change in the factors of safety can also be achieved by not just changing the height of the cross sections, but by using different materials that have a different stiffness than the one used for this model. Also, the maximum displacement at the tip of the structure is decreased from 14.195 in. for the initial design to 2.02 in. for the final design of the bending stress phase. This decrease in deflection also legitimizes the slight increase in buckling margins of safety for the binary links. The ANSYS plot of the displacement of this final design is shown in Figure 15.
With the final design of the bending stress phase, a repeated look at the gravitational analysis is performed as well. This allows for an inspection of the deflection of the redesigned structure under gravitational loads. When analyzed, the design of the structure from the bending stress phase deflects 0.2638 in. at the tip as compared to 2.12 in. from the initial design of the gravitational analysis. The structure is designed to be more lightweight and structurally efficient and tailored for aerodynamic loads which it may encounter.

Analyses are also performed on the fully and half deflected positions. These tests use the final design that is achieved from the successive analysis approach of the flat case. The fully and half deflected cases are used as more of a check for the displacement under gravitational loads or for buckling or bending stress failure. The results show that the structure does not fail in the half and fully deflected positions. Verification calculations of the bending stresses and the results of a simplified ANSYS model are performed.

### C. Model Verification Calculations

A sanity check of the bending stresses produced by ANSYS is performed. The check of the bending stresses is found by dividing the maximum bending stresses by the actual bending stresses. The maximum bending stresses are found mathematically through,

\[
\sigma_{\text{max}} = \frac{Mc}{I} \tag{8}
\]

where \( M \) is the applied bending moment, \( c \) is the distance from the neutral axis to the top or bottom outer fiber of the beam, and \( I \) is the area moment of inertia of the beam cross section. The applied bending moment for each beam is found from the ANSYS model. If the stresses check, they should be close to a value of 1.

The results show that the design of each link is approximately equal to 1.0 and the final design checks.

ANSYS model verification finite element calculations are performed in Wiggins. A simplified model of the first section of the mechanism without gravity effects is created as shown in Figure 16. The resulting force in the binary member from ANSYS is 325.28 lbf. Compared to the calculation of 325.35 lbf, a percent difference of 0.021 is computed for the member force. Also, the ANSYS model gives a compressive axial member stress of 1150.5 psi. The calculation of this stress gives 1150.69 psi making a percent difference of 0.016. The ANSYS model is verified to be accurate for nodal displacements, rotations, forces, and moments with a less than 0.04 percent difference. This confirms not only the accuracy of the model, but also its representation.

### V. Conclusions

A single-degree-of-freedom kinematic mechanism has been presented that achieves the shape change and range of motion of a variable dihedral HECS wing. From a linear aerodynamic analysis performed, predicted load distributions of the wing configuration were determined. With these aerodynamic loads and the gravity loads of the mechanism, a structural analysis will be carried out. This analytical structural study will help determine the critical members of that need to be redesigned.

### Acknowledgements

The authors acknowledge the support of NASA Langley Research Center, Dr. Lucas Horta, grant monitor. The aerodynamic advice of Dr. William H. Mason, the work of Will Whittier, and discussions with Dr. Barry Lazos are also appreciated. We would like to acknowledge the Morphing Wing Group of Virginia Tech.
References


